ADAPTING MATH INSTRUCTION TO SUPPORT PROSPECTIVE ELEMENTARY TEACHERS

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ABSTRACT

Purpose – Elementary teachers’ understanding of mathematics is a significant contributor to student success with mathematics. Consequently, teacher educators are frequently charged with the responsibility of supporting the development of prospective elementary teachers’ mathematics content knowledge as they re-learn concepts in ways they are required to teach. The purpose of this paper is to describe one teacher educator’s efforts to support prospective elementary teachers’ tenuous understanding of rational numbers.

Design/methodology/approach – Given the variety of factors influencing the development of teacher knowledge, a mixed method research design was utilized. Research participants were prospective elementary teachers enrolled in a 9-week elective course who agreed to participate in the study (N=40); while the control group were prospective elementary teachers not enrolled in the elective course (N=35).

Findings – The results of this study indicate that it may be possible to improve prospective teachers’ conceptual understanding of mathematics by providing additional short-term support such as an elective course and/or web-based video clips. However, the program intervention can only build upon the existing knowledge that prospective teachers bring when they begin their Bachelor of Education programs.

Originality/value – For prospective teachers with a limited foundation in mathematics (e.g., less than 4 secondary school mathematics courses), short-term support may be insufficient to compensate for their nebulous understanding of rational numbers. Based on this finding, one-year Bachelor of Education programs might consider, either: including Grade 12 mathematics as a pre-requisite for elementary teacher applicants; or mandating enrolment in a full-year math content course similar to the elective course described in this paper.

KEYWORDS

Elementary teacher education, mathematical knowledge, rational numbers, web-based learning
1. INTRODUCTION

Successful mathematics teachers need preparation that covers knowledge of mathematics, of how students learn mathematics, and of mathematical pedagogy that is aligned with the recommendations of professional societies (National Research Council, 2010, p. 123).

The Conference Board of Mathematical Sciences and the National Council of Teachers of Mathematics are two such professional societies. Both organizations support the assertion that teachers must possess a deep understanding of the mathematics they are expected to teach (Conference Board of Mathematical Sciences, 2001; National Council of Teachers of Mathematics, 2000). Furthermore, Darling-Hammond’s (2000) review of state policies and Hill, Rowan and Ball (2005) seminal study on the effect of teacher knowledge on student achievement broaden this position, concluding that the combined influence of teacher mathematical content knowledge and pedagogical content knowledge more strongly correlate with student achievement than any other moderating factor, including socioeconomic and language status.

The research presented in this paper describes one mathematics educator’s efforts to support prospective elementary teachers’ tenuous understanding of the mathematics they are expected to teach. The purpose of this paper is two-fold: to highlight the effects of a minor program change on the content knowledge of prospective teachers; and to contribute to the growing body of research on the essential understandings for teaching elementary mathematics (Ball, Hill & Bass, 2005; Hill & Ball, 2009; Hill, et al., 2008; Hill, Rowan & Ball, 2005; Shifter, 1998; Shulman, 1986); more specifically, for teaching rational numbers (Jones Newton, 2009; Newton, 2008; Siegler, et al., 2010; Tirosh, 2000).

In 2009, a mathematics elective course was introduced to support prospective teachers that were anxious concerning teaching elementary mathematics. The Math4Teachers elective and web-based video clips (WBVCs) were designed to seek ways, within the time constraints of a Bachelor of Education program, to prepare future teachers who were confident and competent to teach rational number concepts common in the Grade 4 – Grade 6 mathematics curricula. This paper describes the theoretical framework, instructional design, and influence of the program intervention on prospective elementary teachers’ understanding and confidence with rational numbers.

2. THEORETICAL FRAMEWORK

2.1 Knowledge for Teaching Elementary Mathematics

When a teacher doesn’t have a deep understanding and affinity for mathematics … it’s hard to imagine how that teacher will help students understand and get excited [about mathematics]. (Shulman, 2001, p.7)

Research highlights a direct correlation between student achievement in mathematics and their teachers’ understanding of the mathematics content they teach (Burton, Daane & Giesen, 2008; Hill, Rowan and Ball, 2005; Ma, 1999; Schmidt, Houang & Cogan, 2002). Although this connection seems self-evident, the paucity of empirical evidence on effective ways to develop prospective teachers’ knowledge for teaching mathematics has been
highlighted in recent research (Burton, Daane & Giesen, 2008; Berk & Hiebert, 2009; Li & Kulm, 2008; Kilpatrick, Hill & Ball, 2009; Swafford & Findell, 2001). One explanation for the scarcity of this empirical research may be the lack of consensus on the knowledge required for teaching. Initially, Shulman (1986) described the knowledge required for teaching as the interconnection between subject-matter knowledge, pedagogical content knowledge and curricular knowledge. Yet, a decade later, Schifter (1998) bemoaned the absence of research on this topic. She urged researchers and mathematics teacher educators to pursue the question: “What kinds of understandings are required of teachers working to enact the new pedagogy?” (Schifter, 1998, p.57)

Since Schifter’s 1998 call for research, the breadth and depth of teachers’ mathematical knowledge has been explored by Ball and colleagues (Ball, Hill & Bass, 2005; Ball, Thames & Phelps, 2008; Hill & Ball, 2009; Hill, Ball & Shilling, 2004; Hill, Rowan & Ball, 2005). Ball et al. have focused on a complex dimension of teacher knowledge, namely, mathematical knowledge for teaching. At the core of mathematical knowledge for teaching is a deep understanding of mathematics content. Specifically, mathematical knowledge for teaching assumes an understanding of “common” mathematics knowledge, which is the content knowledge “that any well-educated adult should have” (Ball, Hill & Bass, 2005, p.22). Unfortunately, many elementary teachers lack this “common knowledge” and, therefore do not have the foundation to build their mathematical knowledge for teaching.

The collective effect of insufficient content knowledge combined with high levels of mathematics anxiety can overwhelm prospective elementary teachers as they begin their careers as mathematics educators. Thus, in 2009, a small, Ontario lap-top university announced the inclusion of a technology–enhanced elective course to support elementary prospective teachers’ struggling with the mathematics content they were expected to teach. The Math4Teachers elective was introduced as an evening course offered in the first semester (August to November) of the program (2 hours/week). In 2010, a series of web-based video clips were created as an accessible resource during practicum placements and beyond the duration of the 9-week elective course.

2.1 Math4Teachers course design

Nurturing students’ understanding of and confidence with rational numbers is one of the most challenging aspects of teaching elementary mathematics (Gould, Outhred, & Mitchelmore, 2006). Unfortunately, this challenge is complicated by teachers’ own conceptual misunderstandings of rational numbers (Jones Newton, 2009; McLeman & Cavell, 2009) which inevitably lead to conceptual misunderstandings for their students. These misunderstandings often follow children into adulthood (Lipkus, Samsa, & Rimer, 2001; Reyna & Brainerd, 2007) and further intensify should these adults pursue careers as elementary teachers (Ball, Hill & Bass, 20005; Ma, 1999; Menon, 2008; Yeping, 2008); thus perpetuating a cycle of misunderstanding.

My challenge, as a mathematics teacher educator, is to disrupt the cycle of misunderstanding by providing prospective elementary teachers with opportunities to re-learn mathematics in ways they are required to teach it. To this end, the Math4Teachers course was designed based on research rooted in effective teaching strategies to support students struggling in mathematics, which include: (1) combining manipulatives and
pictorial representations to model abstract concepts (Bulter, Miller, Crehan, Babbitt, & Pierce, 2003; Gersten, et al., 2009, 2006; Siegler, et al., 2010); (2) incorporating a mixed model of instruction which blends principles of explicit instruction including teacher modeling, guided practice, and corrective feedback (Baker, Gersten & Lee, 2002; Flores & Kaylor, 2007; Gersten, et al., 2009, 2006; Kroesbergen & Van Luit, 2003); and (3) featuring ample time for discussion, including student-focused discussions which provide alternative solution strategies expressed in students’ language (Grouws, 2004; Shellard, 2004). Embedded in the course design are opportunities for prospective teachers to “do” similar tasks as their students in small cooperative groups, as well as discuss the nature of the mathematics pedagogy.

Beyond teaching strategies, the Math4Teachers course is designed within an explicit lesson sequence. The nine-lesson instructional sequence is derived from research which supports the progressive development of conceptual knowledge through a continuum of understanding (Moss & Case, 1999; Small, 2005; Small, McDougall, Ross, & Ben Jaafar, 2006). Specifically, Moss and Case (1999) advocate teaching fractions and decimals through a lesson sequence which builds on student knowledge of rational numbers beginning with benchmark percentage representations, and then connecting percentage to decimal representations and finally connecting decimals to fractional representations.

Accordingly, the Math4Teachers lesson sequence begins with benchmark percentages which can be represented as terminating decimals; and then moves forward to connecting benchmark percentages to concrete representations of decimals using virtual and concrete manipulatives (e.g., 10x10 geoboards, Base Ten Blocks). Approximately half of the instructional time (~ 10 hours) is allotted to representing, comparing and decomposing decimals using manipulatives; and then connecting the concrete models to pictorial representations; and finally, to the abstract symbols. Instruction transitions naturally from decimals to fractions by modeling place value vocabulary for identifying decimals. For example, labeling 0.4 as “four-tenths” not “point four”, or naming 1.03 as “one and three-hundredths” not “one point zero three”. The subsequent fraction lessons progress from representing tenths and hundredths using area models (e.g., geoboards, Base Ten Blocks) to representing and comparing unit fractions and then simple fractions using area models (e.g., geoboards, Tangrams, pattern blocks) and measured models (e.g., fraction strips, linking cubes, Cuisenaire Rods™). The instructional sequence concludes with two lessons on addition and subtraction of fractions; specifically, exploring relationships between symbolic procedures and concrete representations (e.g., common denominator, simplifying fractions).

2.2 Web-based learning tools

Web-based learning tools (WBLTs), such as web-based video clips (WBVCs) evolved from a need for accessible, affordable and flexible learning via the Internet (Ally, 2004; Downes, 2004). WBLT are distinct from other digital resources in that “instructional design theory … must play a large role in the application of WBLTs if they are to succeed in facilitating learning” (Wiley, 2000, p.9). Kay and Knaack’s (2005) review of the WBLT literature, categorized the many WBLT definitions as either technology-focused or learning-focused. They, in turn, defined WBLTs as “reusable, interactive web-based tools that support the learning of specific concepts by enhancing, amplifying, and guiding the
cognitive processes of learners” (Kay & Knaack, 2005, p. 231). The WBVCs discussed in this paper adhere to Kay and Knaack’s (2005) definition of WBLTs.

Given the ubiquitous computing environment at this lap-top university, the development and use of WBLTs have been the focus of various research studies within our Faculty of Education (Kay & Kletskin, 2010; Kay, Knaack & Muirhead, 2009; Kay & Knaack, 2009a; Kay & Knaack, 2009b; Kay & Knaack, 2009c). Consequently, the design and implementation of the WBVCs discussed in this paper are grounded in sound theory and practice both in the relevant literature and in other examples of implementation within the university. For example, the WBVC content and design were guided, in part, by Kay and Knaack’s (2007) findings on conditions that most benefit student learning via technology, including student perceptions of the usefulness of the content, clear instructions, and visual appeal.

Thus, the WBVCs described in this article are easy to navigate, have visual appeal and explore content considered useful to the end-user. More importantly, the WBVCs begin to address the challenge of limited face-to-face instructional time common in prospective education programs. By grounding the content and instructional sequence of the WBVCs in the same research as employed in developing the Math4Teachers course (Bulter, et. al., 2003; Moss & Case, 1999; Small, 2005); integrating the two modes of delivery was unproblematic. Moreover, the WBVCs afford prospective elementary teachers greater flexibility in learning; as they can access the online videos in a “just in time” manner. As such, the WBVCs may be a viable instructional strategy for providing additional support to prospective teachers struggling with mathematics.

For the purposes of this article, only five Representing Decimals WBVCs are discussed: Decimal vocabulary; Comparing decimals; Exploring tenth; Using geoboards to represent decimals; and Exploring hundredths on the geoboard.

3. RESEARCH METHODOLOGY

National Research Council asserts that

Both quantitative and qualitative data about the programs of study in mathematics offered and required at teacher preparation institutions is needed, as is research to improve understanding of what sorts of preparation approaches are most effective at developing effective teachers (2010, p. 124).

In view of this assertion and the array of potential factors influencing the development of teacher knowledge, both quantitative and qualitative data were collected. Specifically, a mixed method research design (Creswell & Clark, 2007) was utilized to triangulate all data collected at the start and end of the academic term. Research participants were prospective elementary teachers enrolled in the 9-week Math4Teachers elective course from August to November, 2010 who agreed to participate in the study (N=40). The control group were prospective elementary teachers not enrolled in the Math4Teachers elective course who agreed to submit their pre- and post- Elementary Mathematics Diagnostic results for analyses (N=35).

3.1 Data sources
Evidence on the value of the WBVCs was based on two homework tasks requiring prospective teachers to view and then comment on the five Representing Decimals WBVCs. Subsequent to viewing the video clips (Total viewing time = 20:14 minutes), teachers reflected on their “most significant learning” and their “enduring questions”. At the end of the term, 40 of the 57 prospective teachers enrolled in the Math4Teachers course re-submitted their reflections for analysis. The written reflections were transcribed into the qualitative data analysis program, ATLAS.ti. The software was used to process data, create codes, and analyze and interpret codes through searching for common words, phrases and themes. The WBVCs are available at http://lesage.blogs.uoit.ca/?page_id=30.

Evidence on the combined effect of the WBVCs and Math4Teachers elective course was based on pre- and post- achievement data from the Elementary Mathematics Diagnostic Tool administered in August 2010 (prior to the Math4Teachers course) and March 2011 (at the end of the program). The diagnostic tool includes 17 multiple choice questions focused on either Number Sense (N=12) including Rational Number questions (N=7); or Number Sense and Measurement combined (N=5). With the exception of two questions created by the author, all questions were adapted from two existing instruments: the Content Knowledge for Teaching Mathematics Measure (CKT-M) created at the Learning Mathematics for Teaching Project at the University of Michigan (Hill, Shilling & Ball, 2004); and the Ontario standardized Assessment of Reading, Writing and Mathematics, Grade 6 (EQAO, 2005; EQAO, 2007). Table 1 summarizes the Elementary Mathematics Diagnostic Tool questions by strand, construct and source.

[Insert Table 1. Elementary Mathematics Diagnostic Tool strands, constructs and sources]

Reliability of Elementary Mathematics Diagnostic Tool Variables

Table 2 shows the Cronbach’s alpha for the achievement variables in the study. Cronbach’s alphas of .70 or greater are considered reliable. Consequently, the Pre- and Post- Elementary Mathematics Diagnostic Tool and two of the three subscale constructs measured achievement as were expected; hence, are internally reliable. However, the Number Sense + Measurement construct reliability was below the 0.70 threshold; thus, was not used as an achievement measure in this study.

[Insert Table 2. Reliability of Elementary Mathematics Diagnostic Tool]

3.2 Results

3.2.1 The combined influence: WBVCs + Math4teachers elective course

Paired sample t-tests and correlations were conducted on Elementary Mathematics Diagnostic Tool scores and subscale construct scores by test occasion. The results indicate
that prospective elementary teachers’ understanding of number quantity increased during the 8-month teacher education program, for both the elective and non-elective group. That said, the mean difference in diagnostic scores for the elective group was significantly greater than the non-elective group. The descriptive and inferential statistics summarizing participant numbers, means, standard deviations, mean differences, correlations and t-tests (significance) for diagnostic scores as a function of elective course enrollment are displayed in Table 3.

[Insert Table 3. Diagnostic scores by test occasion as a function of elective course enrolment.]

The results indicate that when data are controlled by elective course enrolment, notable patterns emerge. Specifically, prospective teachers enrolled in the elective course not only saw greater improvements in their mathematical content knowledge than those not enrolled in the elective course; but they also retained this knowledge five months beyond the end of the elective course (post-test). The analyses reveal statistically significant increases in Mean Difference scores of:

- Mathematical content knowledge (Total Diagnostic score)
  - Elective Group +12.2% versus Non-elective Group +6.5%;
- General Number Sense understanding
  - Elective Group +14.2% versus Non-elective Group +7.1%; and
- Rational Number Sense understanding
  - Elective Group +13.0% versus Non-elective Group +5.7% (not significant).

Thus, prospective elementary teachers who opted to complete the 8-week elective course increased their post-evaluation scores by approximately double that of those not opting to enroll in the elective course (i.e., M.D. Elective group = 2 × M.D. Non-elective group for each evaluation construct). Moreover, enrollment in the Math4Teachers elective strongly correlates with Post-Diagnostic test scores (r=.742); thus, accounting for 55% of the variance in Post-Diagnostic test scores.

To better understand the influence of other factors on the achievement outcomes of the Elective group, Pearson correlations were generated between three independent variables (final High School math course, number of university math courses, and age) and the six mathematical knowledge variables (pre-post Diagnostic scores, General Number Sense Subscale and Rational Number Sense Subscale). As illustrated in Table 4 significant correlations among the study variables within the Elective group, include:

- **Age** correlates with **final High School math course completed** (r=.423). That is, more advanced secondary school mathematics courses were completed by mature students. Younger participants (21–30 years old; N=31) terminated their mathematics studies after completing Gr.11 Academic Math; while mature participants (41–54 years old; N=5) concluded their mathematics studies after completing Gr.12 Academic Math.
- **Age** also correlates with **Pre-Diagnostic test scores** (r=.348). Specifically, mature (41–54 years old) participants’ mean Pre-Diagnostic scores (69.4%) where
statistically greater than the mean Pre-Diagnostic scores (51.4%) of the youngest (21–30 years old) participants.

- The final High School math course completed correlates with each of the significant mathematical knowledge variables in the study. Specifically, the level of High School math correlates with performance on the Pre-Diagnostic test (r=.505) including the two Number Sense Subscales. The correlation, though significant, decreases on the Post-Diagnostic test (r=.370).

- Pre-Diagnostic test scores strongly correlate with each of the other significant knowledge variables. Specifically, Pre-Diagnostic test scores correlate with Pre-General Number Sense scores (r=.884); Pre-Rational Number Sense scores (r=.800); Post-Diagnostic scores (r=.702); Post-General Number Sense scores (r=.674); and Post-Rational Number Sense scores (r=.651).

- The two Pre-Number Sense subscales strongly correlate with their respective post-subscals. Specifically, Pre-General Number Sense scores correlates (r=.681) with Post-General Number Sense scores; while Pre-Rational Number Sense scores correlates with Post-Rational Number Sense scores (r=.613).

[Insert Table 4. Correlations showing relationships among statistically significant variables]

In summary, the combination of the WBVCs and enrollment in the Math4teachers course positively influenced prospective elementary teachers’ General Number Sense and Rational Number Sense. The findings highlight Pre-Diagnostic test scores (including both subscales) as the single best predictor of Post-Diagnostic test scores. The findings also highlight a significant correlation between the final high school math course completed and the depth of mathematical knowledge of prospective elementary teachers. However, the level of high school mathematics did not correlate with the change in mathematical knowledge from pre- to post-evaluation (e.g., the mean differences in Diagnostic test scores and subscale scores). Thus, although the intervention positively influenced participants’ knowledge of numbers, it did not compensate for the gap in fundamental number sense commonly acquired during high school mathematics coursework.

### 3.2.2 Influence of the Web-based video clips

Evidence concerning the value of the WBVCs was founded on two homework tasks requiring pre-service teachers to view and then comment on the five Representing Decimals clips. Subsequent to viewing the video clips (Total viewing time = 20:14 minutes), prospective teachers reflected on their “most significant learning” and their “enduring questions”. At the end of the term, volunteers re-submit their written reflections for research purposes. Of the 57 prospective teachers enrolled in the Math4Teachers course, 40 re-submitted their reflections for analysis. The reflections were transcribed into the qualitative data analysis program, ATLAS.ti. The software was used to process data,
create codes, and analyze and interpret codes through searching for common words, phrases and themes.

Based on the analyses of the participants’ reflective comments, the Representing Decimals WBVCs were positively received. The following three themes emerged from the data analysis:

- Mathematical content knowledge + efficacy;
- Pedagogical content knowledge + efficacy; and
- Instruction design considerations that contribute to learning.

**Mathematical content knowledge + efficacy**

With the exception of six participants, the prospective elementary teachers indicated the WBVCs influenced their content knowledge of rational numbers. For many (N=21), the WBVCs served as a “refresher” of previously “forgotten or never really understood” knowledge; while for others (N=13), the WBVCs offered new information that they had not previously known or misunderstood. The participants identified three domains of improved content knowledge: decimal terminology, face value versus place value, and comparing decimal quantities.

**Decimal terminology:** The majority (N=31) of the participants identified learning new terminology specific to decimals. In particular, participants misunderstood that the word “and” denotes a decimal; that the word “point” should not be used when naming a decimal (e.g., 3.2 should be read as “three and two-tenths” not “three point two”); and that the digit furthest to the right dictates the name of a decimal (e.g., 3.04 is read “three and four hundredths” because the place value of the 4 indicates hundredths). Moreover, two participants revealed misunderstanding the significance of “-th” when referring to decimals. One participant stated,

I did not know that we had to emphasize the ‘-th’ sound in the place value system. My previous teachers had never taught me the way I am learning it now and it is very interesting to see how simple it can be.

**Face value versus place value:** Participants (N=6) also indicated that the WBVCs rectified their place value misunderstandings. For example, participants indicated that through viewing the Comparing Decimals WBVC, they understand “it is the place value of the digits that matter not the number of digits”. One participant explains, “I thought 0.0948 was greater than 0.13. Unlike whole numbers, it is not how many digits you have (i.e., 948 versus 13), but their place value”. Another participant expressed relief, stating,

... I could never picture [for example] a number written to three decimal places because I always said ‘point four three two’ (0.432). I was hearing the ‘hundred’ and perhaps picturing the whole number; so I pictured a hundredth decimal number. I was confused about where the ‘thousandths’ came in. ... [The WBVC] has now fused my imagined numeral with the verbal and visual representation.

Finally, one participant’s comment radically influenced my awareness of the limitations in mathematical knowledge of some prospective elementary teachers in the Math4Teachers course. She states, “I now know that the first
position behind the decimal is called ‘tenths’, I would have called it the ‘ones’ position’.

**Comparing decimal quantities:** Although many participants (N=20) expressed interest in viewing the comparison between 0.2 and 0.20; most (N=14) focused attention simply on the decimal name not the conception of decimal equivalence. For example, a participant explains, “I would have pronounced these two numbers the same, not counting the zero. Now I have learned the proper way is pronouncing the second number as twenty-hundredths.” By contrast, other participants commented on developing a new understanding of the relationship between fractions, decimals and visual representations. For example, one participant explains,

[The WBVC] helped me understand fractions as well as decimals. The part of clip on 0.20 and 0.2 being hundredths and tenths made sense to me. I always knew this, but I don’t think that I really understood why.

Another participant asserts,

When I saw the clip on this, it was as if a light bulb went on in my head! I always knew that 0.20 and 0.2 were the same, but it really makes a difference when you can see that 0.20 is twenty hundredths and 0.2 is two tenths on the Geoboard!

Interestingly, although the majority of the participants (N=34; 85%) indicated that the WBVCs substantively influenced their understanding of rational numbers, only a few (N=7; 18%) explicitly cited influences on their mathematics self-efficacy or improved confidence in their abilities to do mathematics. Participants offered general comments concerning increased mathematics self-efficacy, such as; “After seeing these five short videos, I already feel more confident in my understanding of decimals …”

However, one participant’s comment is worthy of sharing as it illustrates the potential impact of learning tools, such as the WBVCs, on students’ confidence and self-esteem:

I am really surprised at how well I grasping decimals. I remember this as one of my worst math experiences; which usually ended in a lot of tears. But, watching the clips and using the manipulatives just made something click.

**Pedagogical content knowledge + efficacy**

Perhaps not surprising, all 40 of the participants indicated the WBVCs influenced their understanding of how to teach rational numbers. However, the purpose of the WBVCs was to provide explicit, teacher directed, just-in-time instruction on concepts deemed difficult for students struggling with mathematics. Thus, although the sequence of five Representing Decimals WBVCs is developmentally appropriate, illustrates decimals using various manipulatives (e.g., Ten frames, Base-Ten Blocks, Geoboards), and demonstrates multiple representations of rational numbers (e.g., visual, verbal and abstract); the WBVCs are not exemplary models of reform oriented mathematics teaching practices. Unfortunately, given the feedback, some participants (N=8) extrapolated the explicit instruction model in the WBVCs as exemplary classroom practice. For example, one
participant concluded, “I will definitely be applying these video clips to teach my future students, as it made [the content] very clear and straightforward”. While another participant commented, “I think that watching the videos was a good reminder to speak slowly when explaining a topic to children”.

In spite of this unanticipated learning outcome, the majority of the pre-service teachers (N=32; 80%) generalized beyond the explicit instruction modeled in the WBVCs and gained new pedagogical insights into reform mathematics teaching practices. Specifically, participants emphasized the value of the following:

- encouraging multiple representations, including variability in the materials used to explore the same concept;
- focusing instruction on progressive developmental of concepts by encouraging concrete models, pictorial representations, verbal descriptions, and abstract symbols / numeric representations;
- providing students with sufficient time to explore concepts using concrete materials;
- utilizing diverse teaching strategies;
- incorporating technology (e.g., interactive whiteboard technology, virtual manipulatives, WBVCs); and
- continued professional development and accessing on-line resources (e.g., WBVCs).

Coupled with an increased understanding of effective strategies for teaching decimal numbers, all 40 prospective elementary teachers indicated that the WBVCs influenced their teaching efficacy. The participants cited improved confidence in how to teach rational numbers. Participants offered comments such as:

I now feel more confident in introducing decimals while paving the way for the young learners to explore ways of interpreting and estimating decimals…

**Instruction design considerations**

In addition to the cognitive and affective outcomes described, the study participants emphasized the significant influence of the WBVC format on the quality of their learning experiences. Specifically, the participants highlighted the following components as contributing to their understanding of the concepts presented in the WBVC:

- integration of the WBVCs into the Math4Teachers course design;
- careful sequencing of the content (e.g., each clip explored one component of the broader concept);
- clarity of the explanations including step-by-step explanations;
- combined use of visual models/virtual manipulatives + abstract representation (numbers) + clear verbal explanations;
- abbreviated viewing time (e.g., each clip was less than 5 minutes in length);
- slower pace than actual classroom lessons;
- ability to control the pace of the learning (e.g., pause to take notes, rewind to review);
- the inclusion of practice questions; and
- the absence of judgement (e.g., pause or rewind the video as often as needed without the judgement of others).
Although the participants deemed the WBVCs an effective tool for improving their understanding of representing decimals; there was general consensus that additional WBVCs are needed to support prospective elementary teachers’ understanding of and confidence with rational numbers. The participants offered the following ‘next-step’ suggestions:

- provide additional examples within each WBVC;
- include other manipulatives for representing decimals (e.g., money, relational rods, fraction strips);
- create additional practice questions for each WBVC;
- include sample classroom vignettes for some of the WBVCs;
- discuss common misconceptions associated with teaching and learning rational numbers; and
- create WBVCs modelling ineffective teaching strategies for teaching rational numbers.

4. CONCLUSIONS

WBVC applications and considerations

The WBVCs discussed in this paper were developed to address challenging content areas; provide prospective elementary teachers with accessible and flexible learning opportunities; and offer additional support for previewing or reviewing important concepts addressed during face-to-face instruction. These instruction design considerations allow prospective teachers more control over their learning experiences. For elementary teachers with a history of negative math experiences; being in control of mathematics is a novel yet welcome change. Thus, providing on-line resources which can be accessed in a “just-in-time” manner seems to be a promising strategy for supporting the individual learning needs of elementary teachers.

By leveraging the use of software such as Camtasia Studio 6 or Smart Notebook to create WBVCs, it may be possible for other university faculty members to maximize instructional time by integrating similar technologies into their teaching practices. Although the paper addresses a very specific university student population, the application would be feasible for other college or university instructors as they could address particularly challenging content areas by creating and integrating WBVCs into their course design.

Program intervention considerations

The results of this study indicate that it may be possible to improve prospective elementary teachers’ conceptual understanding of mathematics by providing additional short-term support such as an elective course and/or WBVCs. However, the program intervention can only build upon the existing knowledge that prospective elementary teachers possess when they begin the program. The study findings seem to indicate that the foundation for rational number sense concepts is constructed during elementary and high school mathematics coursework. Specifically, the results indicate a significant correlation between the final high school math course completed and the depth of mathematical knowledge of prospective elementary teachers.
Thus, for prospective elementary teachers with a limited number of high school mathematics courses (e.g., less than 4 courses), this type of intervention or additional support may be insufficient to compensate for the gap in fundamental rational number sense acquired during high school mathematics coursework. Based on this finding, one-year Bachelor of Education programs might consider, either: including Grade 12 mathematics as a pre-requisite for elementary teacher applicants; or mandating enrolment in a full-year math content course similar to the Math4Teachers elective. The intervention described in this paper, however, suggests an alternative strategy for modifying an existing Math Methods course or offering an elective course for teachers struggling with mathematics.

This research offers insight into better understanding the program components and lesson sequence which may influence the development of elementary teachers’ knowledge of rational numbers, and in turn, their efficacy as mathematics teachers and learners. Although this research is in the early stages and much work lies ahead; the immediate research goals include: (1) expanding the rational number WBVCs based on the participants’ suggestions; (2) expanding the WBVCs to include measurement concepts deemed problematic; (3) modifying the Math4Teachers course based on current data analysis; and (4) continuing to monitor the impact of ongoing program changes.

The ultimate research goal is to determine the breadth and depth of mathematical knowledge necessary to teach rational numbers at the elementary level and provide appropriate learning tools, such as WBVCs, to assist other Faculties of Education support their prospective teachers in developing this knowledge. When this goal is attained, we may be able to substantively impact the mathematical learning experiences of future generations of children in our elementary school systems.
REFERENCES


