Completed by Nina Mistry and Golsa Faghani
Sections 05

Geometry and Spatial Sense

Diagnostic Assessment

Grade Level: Appropriate for grades 4 -6

Assessing the Students' Levels of Thinking Based on the van Hiele Model

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The Rationale

The Van Hiele Theory

According to the van Hiele theory (1986), with appropriate instructions, there are five distinct levels of understanding through which students progress in relation to geometry (Crowley, 1987). A student could demonstrate geometric thinking that fits into any of the Van Hiele levels, at any age. The development of geometric thought based on this model, was created in the 1980s by two Dutch middle school teachers and researchers, Dina van Hiele-Geldhof and Pierre Van Hiele. The Van Hiele levels begin with the most basic component— visualization which is the ability to see geometric shapes as a whole without focusing on their particular attributes, and continue to the most advanced level— rigor, which is the ability to learn that geometry needs to be understood in the abstract (Van Hiele 1999). This model is unrelated to the children's developmental skills or stages of thinking where students must reach a certain age to progress through the levels; rather, it is related to the individuals' prior experiences through activities and opportunities (Van Hiele 1999). In other words, students graduate from their current levels to the next one through practice rather than age, and it is crucial for teachers to provide experiences and tasks that allow students to develop accordingly. It is the teacher's responsibility to maintain awareness of each individual student's Van Hiele level of thinking through proper observation and record keeping. The teacher can then design geometry tasks to match the student's level and eventually raise the child's thinking to a higher level.

How to Apply the Theory

Through the proper implementation of the Van Hiele theory, the teacher will be provided with an opportunity to familiarize his/herself with the students' strength and areas of improvement. The teacher will then be able to facilitate a student's achievements toward further progress by asking questions at the next higher level. According to Mason (1997), using lecture and memorization as the main methods of instruction will not lead to effective learning.

Teachers should provide their students with appropriate experiences and the opportunities to discuss them (such as using manipulatives or group work). In addition, as explained by Mayberry in *The van Hiele Levels of Thought in Undergraduate Preservice Teachers*, "The teacher should provide experiences based on the phases of learning to develop each successive level of understanding because if a teacher teaches a level of thought that is above the students' understanding, the student will fail to comprehend the content". If a student is being taught a content that is above their level of comprehension, it is likely that he/she will try to memorize the material without truly mastering the content. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation. When teaching mathematical concepts, teachers must maintain careful consideration of the students' progress and apply differentiated instructions depending on the skills of the students within the class in order to lead them through the necessary experiences to move on.

The Assessment

According to the Van Hiele model, because the phases of learning are recurring, they do not necessarily correspond to certain types of assessment thus teachers need to strive to make assessment an ongoing, daily aspect of the teaching process, rather than an occasional interruption. The following assessment is one which could be administered as a diagnostic, formative or summative to determine the students' geometric level of thinking. This assessment is appropriate for the junior grades as the questions only range from levels 0 to 2 of the Van Hiele's model of thinking. Furthermore, this assessment is "one on one" rather than "pencil paper" which creates the opportunity for the use of manipulatives, as well as the assessment of the students' thought process. For instance, the second question at station one requires the students to distinguish between the symmetrical and non symmetrical shapes, which fits into the level 0 – visualization category, which is the most basic level and means the student can distinguish a shape by its appearance. For students who successfully give the correct response to the question and show that they have mastered the level 0 thinking model of Van Hiele, they will then be prodded forward by being asked level one questions such as " How do you know shape N is not symmetrical?", encouraging the use of descriptive thinking.

Asking the student to explain how they knew a certain mathematical concept, could pave their way to make certain logical claims. While some students may describe the properties of shapes using level 1 thinking, gifted children might be pressed to classify a set of shapes on the basis of these properties or to deduce ideas and relationships about the shapes (Geddes and Fortunato, 1993). It is reasonable to assume that such children are likely to develop their own definitions of certain shapes or to deduce relationships between some shapes such as a triangle and a quadrilateral. In conclusion, such assessments are needed to provide insight into how students understand definitions and how such definitions are used in making and supporting their claims.

The following assessment and the questions that the students will be asked at the two stations, support the van Hiele model of student geometric thinking.

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Station One (Interview Sheet)

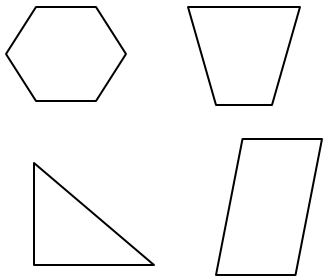
Big Idea: Geometric Properties

At this station, the students will be assessed on their ability to identify symmetry and congruency in two dimensional shapes as well as their ability to identify and sort different quadrilaterals based on their geometric properties.

Materials:

Student Worksheet, Pencil, Pattern Blocks, Chips/Marker

1. The students should be assessed on their ability to identify various geometric shapes and their properties. **Ask:** What are the names of these shapes? **Show:** The student various shapes using pattern blocks and provide students with the names of the shapes separately. **Ask:** Can you match the geometric shape with its correct name? **Extension:** If the student had difficulties with the question, move on and do not ask further questions. If the student was able to identify the shape with its name, they have successfully completed Level 0 (Visualization). **Ask:** How did you know these names matched the shape? **Ask:** These shapes are polygons, what do they have in common?



Words

Hexagon

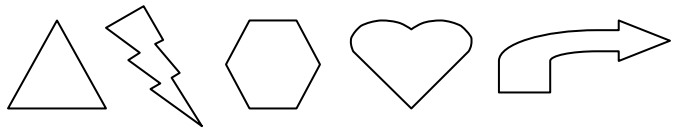
Trapezoid

Right Angled Triangle

Parallelogram

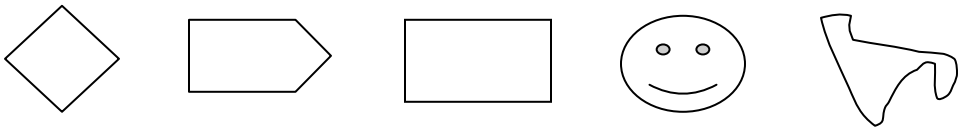
Correct Answer: The students should place the correct words with their matching shapes and be able to say the names of each shape once they have placed them down. **Scoring:** If the student identified the shapes with their names, they will be at Level 0. If the student could explain how they knew the words matched the shapes (i.e. a right angled triangle has three sides and is 90 degrees) the student should receive Level 1 (Descriptive / Analysis). If the student could explain that the shapes are polygons because they are closed, have straight lines, more than one equal side and mention that the Trapezoid and Parallelogram are Quadrilaterals because they have four sides, then the student would receive Level 2 (Informal Deduction).

2. **Show** the student a series of shapes. Ask the student to identify the shapes that are symmetrical by placing a chip on the correct shapes.
   Say: I have five shapes, can you place a chip on the shapes that are symmetrical?
   Extension: If the student was able to choose the correct symmetrical shapes, ask the student "Why did you think these shapes were symmetrical?" if the student could articulate that if the shape is split in half it would be equal/same on each side then the student would receive Level 1.



Correct Answer: The students should have mentioned the triangle, hexagon and heart as being symmetrical to receive Level 0. If the student could explain their reasoning for choosing the three shapes, they will receive Level 1.

3. **Ask** the students to draw the line of symmetry for the geometric shapes shown on their worksheet. **Say:** Can you draw the line of symmetry for each of these shapes? Remember there are can more than one line of symmetry for each shape or there could be no lines of symmetry at all. **Extension:** If the student has trouble drawing the lines of symmetry, do not ask the student any more questions, however if they can complete the task, **Ask:** "why are some shapes symmetrical and others not symmetrical and why did some shapes have more than one line of symmetry?"



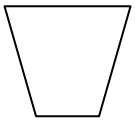
Correct Answer: Observe the students as they should notice that the diamond shape could be drawn with four lines of symmetry. The pentagon has one line of symmetry; the rectangle has four lines of symmetry. The smiley face has one line of symmetry and the freeform shape does not have any lines of symmetry. **Scoring:** Level 0 is if the student could identify lines of symmetry by drawing it on the shapes. Level 1 is rewarded for the student's explanation of how some shapes have more than one line of symmetry, if they said because if you folded the shape in half it would be symmetrical or the freeform shape is not symmetrical because it does not have any equal sides.

4. **Ask:** The students to draw a symmetrical figure on the left side of the line and then create its symmetrical figure on the right side. This activity will confirm if the student understands what a symmetrical figure should look like. **Say:** You get to create your own symmetrical shape! Can you draw half of the shape on the left side of the line and its symmetrical half on the right? Remember, it has to be a symmetrical shape. **Extension:** If the student is successful, give the student the opportunity to explain why each side is symmetrical. **Say:** How do you know both sides of your shape are symmetrical?

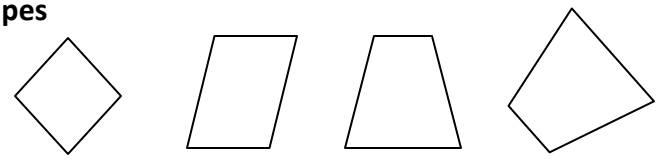
Correct Answer: The shapes will vary for each student. However, if the student was not successful in creating a symmetrical shape because they needed excessive clarification or probing, they will not be asked further questions. If the student could explain that they made sure each side of their shape was the exact same as the other side (same length or size) then they will receive Level 1 because they explained how they knew it was symmetrical. If the student could mention geometric relationships such as knowing that parallel lines or straight lines are useful for creating symmetry then they would receive Level 2 due to the ability to make the connection with other geometric terms and understanding the relationship.

5. The students will now be tested on congruency as they will need to identify the shapes that correspond with Figure A. Have pattern blocks placed in front of the student, they will identify by placing the congruent shapes below Figure A. **Ask:** Can you show me which shape is congruent with Figure A? Remember, there could be more than one answer here. **Extension:** "Why did you think these shapes were congruent with Figure A"? **Look for:** If the student is able to mention that the shape is the same shape, same size or equal to figure A. In addition, if the student could use the process of elimination by comparing the length and width of the parallelogram and diamond and mention congruency means the same shape but different position, then they would receive Level 1 because they have named properties and attributes.

Figure A



Shapes

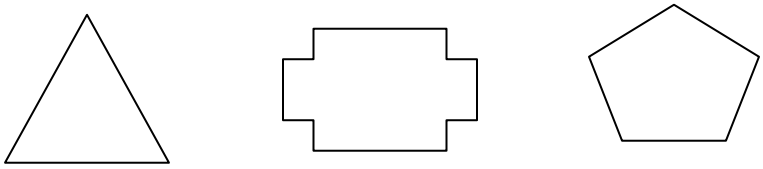


Correct Answer: The student should have placed both the third and fourth Trapezoid under Figure A. **Scoring:** If the student placed the incorrect shapes under the figure move on to the next question. If they placed the correct shapes under the figure but could not explain why then they would receive Level 0. If the student could explain how they knew the third and fourth shape was congruent then they should receive Level 1.

6. This question should be asked only if the student received Level 1 on question #5. Show the student the two right angled triangles on their worksheet. **Ask:** Can you identify the congruent angles on each right angled triangle? **Extension:** Ask the student how they knew which angles were congruent.

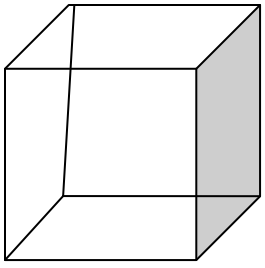
Correct Answer: See if the student can explain that the congruent angles are flipped and it can be determined by looking at the lines. The student should receive Level 1 if they could analyze the triangles and mention how the lines helped indicate which sides were the same lengths.

7. If the students scored Level 1 on question #6. **Ask:** Students to identify various attributes of two dimensional shapes. **Ask:** How many sides does this shape have? How many angles? How many vertices? **Show:** The students the shapes on their worksheets, ask the students to write the number of sides, angles and vertices.



Correct Answer: There are 3 sides on the Triangle, 12 sides on the Cross, 5 sides on the Pentagon. There are 3 vertices, 12 vertices and 5 vertices. If the student could explain how they counted the sides then the student will receive Level 1. If the student could do the bonus with an explanation for their answer, that would be Level 2.

Bonus



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Station Two (Interview Sheet)

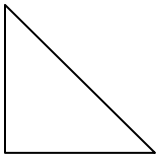
Big Idea

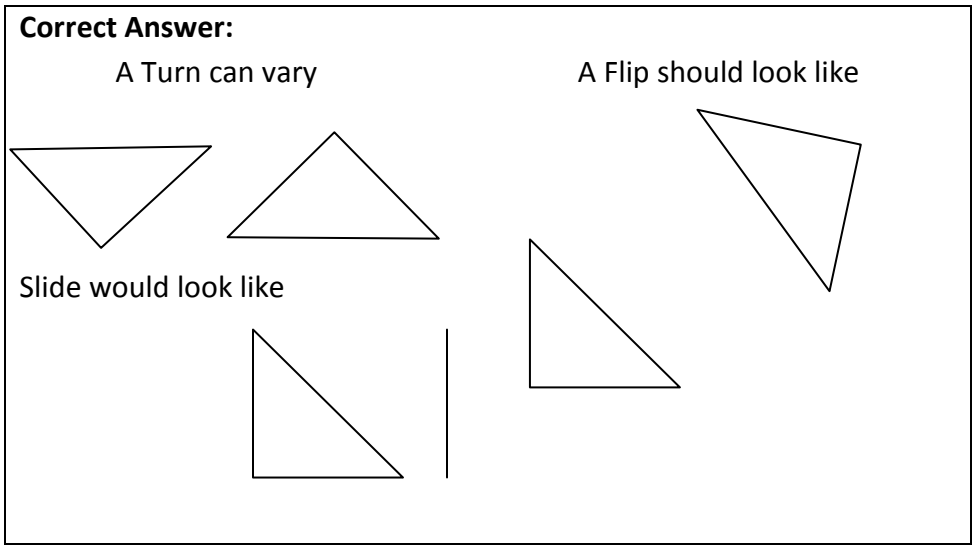
At this station the students will be assessed on their knowledge and comprehension of Location and Movement. The students will be asked to identify shapes in various transformations such as a reflection, rotation, turn, flip or slide.

Materials

Student Worksheet, Pencil, Tanagram pieces, Geoboard, Elastics

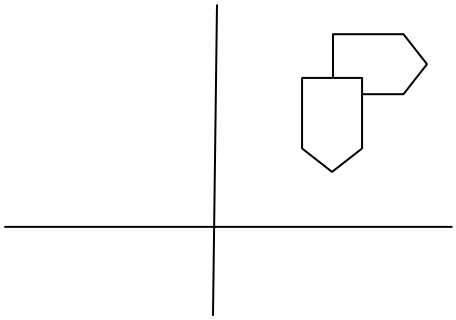
1. **Show** Tanagram pieces and **Ask** the Student to change the initial position of the geometric shape, by either turning, flipping or sliding the shape. **Say:** Can you show me a turn, flip and slide of this shape? **Extension:** Ask the student how knew how to make a turn, flip and slide. **Ask:** Does a turn, flip and slide remain you of another type of transformation and why?



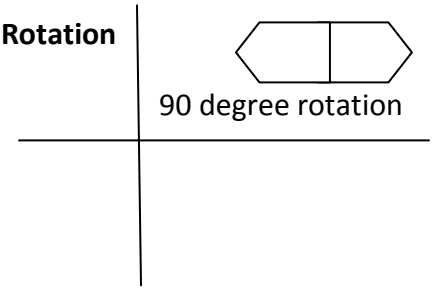


Correct Answer: If the student does not show the correct transformation but could not explain how they knew, the student will receive Level 0, do not ask further questions. If the student can describe that a turn requires the shape to stay in the same position and move from left to right, they would receive Level 1. If the student could compare a turn with a rotation or a flip with a reflection that would display higher thinking and the student would receive Level 2.

2. Using a Geo board, the student will have a chance to demonstrate their understanding of turns. **Ask** the student to make a turn of the pentagon below using a Geo board. **Say:** Can you rotate this shape on the Geo board using the elastics?

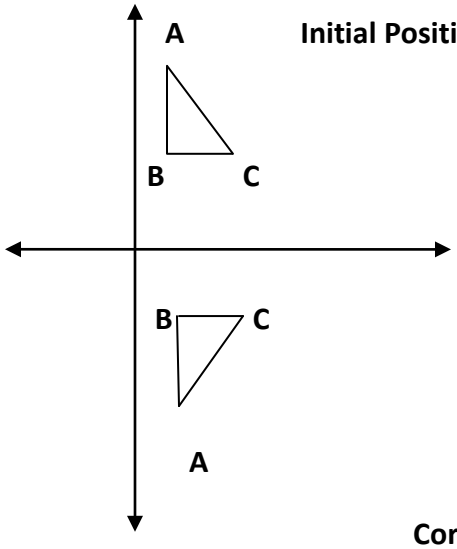


Correct (Answer) is a Rotation



If the student does not create the correct rotation move on to the next question, if the student could not explain how they knew how to make a Rotation the student would receive Level 0. If the student was able to rotate the shape and explain how they knew what to do, score the student with a Level 1.

3. **Show** the students a picture of a shape in the initial position and then in its moved position. **Ask** the students to identify what movement the following shape is in. **Say**: The Triangle in the Initial Position" has moved. What type of movement is it in now?



Initial Position

Has the triangle moved into a

a) Slide

b) Turn

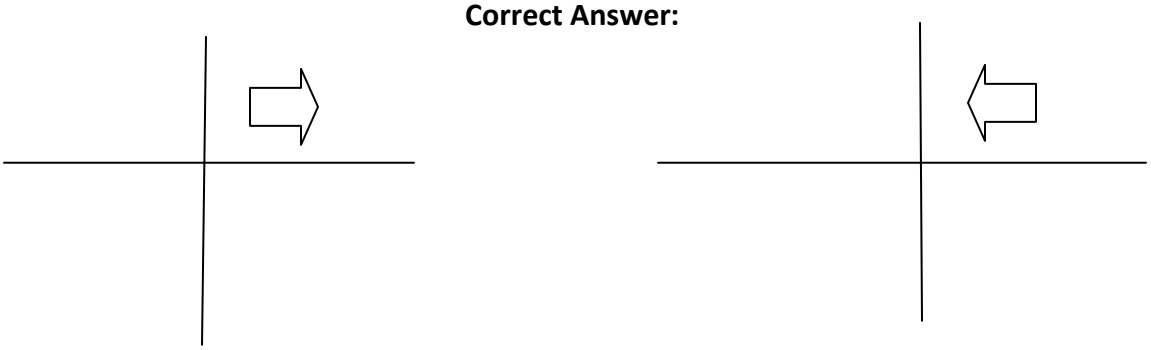
c) Reflection

d) Flip

Correct Answer: Answer: Reflection. If the student could not explain why they chose the Reflection the student will receive Level 0. If the student could explain that the shape is a reflection because it crossed the grid and every point is in the same position except **A** then the student will receive Level 1.

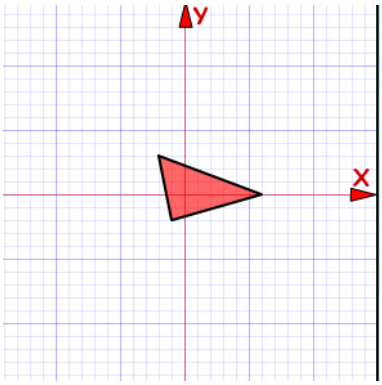
4. If the student was able to answer question #2 without hesitation, **Ask:** the student to do this advanced question. **Say:** Can you draw this shape again in a 180 degree angle to the right? **Extension:** Ask how do you know this is correct?

Correct Answer:



Scoring: The student will have to know the difference between 90, 180, 360 degrees. A Level 0 is when the student could not explain how they get the correct answer because they could have guessed the position. If the student could give reasoning with a drawing of angles, showing they knew what 90, 180 degrees looked like then give the student Level 1 for their analysis.

5. Using grid paper, **Ask:** The student to draw a Translation for the triangle. **Say:** Can you draw a translation for the triangle on the grid? What did you have to remember when drawing the translation? Could you draw the translation 3 unit up?



Reference: http://www.mathsisfun.com/geometry/translation.html

Correct Answer: The answer will vary depending on if the student draws the translation up or down on each side. However, if the student was able to successfully draw the translation, the student will receive Level 0. If the student could explain that translations move in the direction and all points are the same distance then the student should receive Level 1. If the student can draw a translation 3 units up correctly and explain then they will receive Level 2.

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Scoring System for Students Assessment

Instead of a traditional rubric, the students will be assessed using *The van Hiele model of Geometric Thought* because it is a more thorough and individualized assessment of the student's knowledge and understanding. This type of assessment is beneficial for both the teacher and the student because it is one on one. Although, it would seem more practical to do a paper pencil test with the entire class, the van Hiele assessment will provide the teacher with a deeper understanding of the student's ability, strengths and areas of weakness.

The student will benefit from this type of assessment because they will have a chance to use manipulatives. Each question will be written down for the student to read over and each question will have a task that requires the student to draw, use manipulatives or observe pictures or numbers. "When students investigate and explore polygon properties rather than focus on recalling rigid definitions, they are more likely to develop analytical skills and geometric critical thinking skills" (Ministry of Education,23). On a "paper pencil" test, the student could achieve high marks by fluke, but this summative assessment is a better approach as the teacher can spend time observing and asking the student questions.

Modifications, Extensions and Considerations

It is important for a teacher to make the necessary modifications, extensions and considerations while conducting the assessment. The students that are comfortable with the material should have the opportunity to try advanced questions to give a fair evaluation. The teacher will be able to see which level the student can reach in the van Hiele's scale. Some students will be at different levels but that is quite normal. After each question, the student will be asked further questions if the student reaches the next level. For example, if a student has answered a question that is at level 0, ask the student how they know, to see if they can answer level 1 then if they are able to answer that question ask a further question to see if the student is able to reach level 2. Although, the student may not be able to answer the question, allowing the student to have a chance to experience higher level thinking is a benefit for the teacher and student to see the capability of the student. The teacher should make considerations for each student while conducting the assessment. Having visuals as well as verbal and written components are essential for each student and it provides every student with a fair chance of doing well on the assessment.

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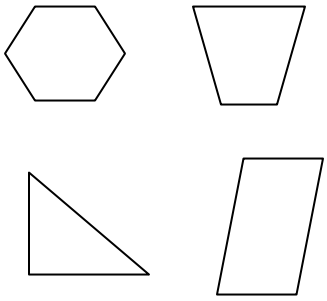
Student Worksheet for Station One

1. The students should be assessed on their ability to identify various geometric shapes and their properties. **Ask:** What are the names of these shapes? **Show:** Show the students various shapes and provide students with the names of the shapes. **Ask:** Can you match the name of the geometric shape with its correct shape?



Words

Hexagon

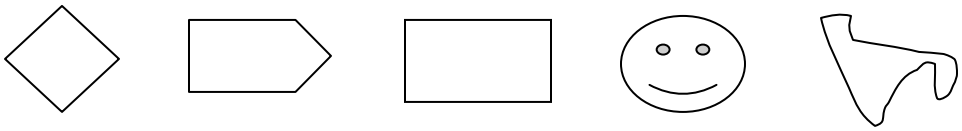
Trapezoid

Right Angled Triangle

Parallelogram

2. Can you place a chip on the shapes that are symmetrical?

3. Can you draw the line of symmetry for each of these shapes? Remember there are can more than one line of symmetry for each shape or there could be no lines of symmetry at all.



4. You get to create your own symmetrical shape! Can you draw half of the shape on the left side of the line and its symmetrical half on the right? Remember, it has to be a symmetrical shape.

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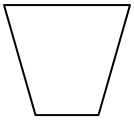
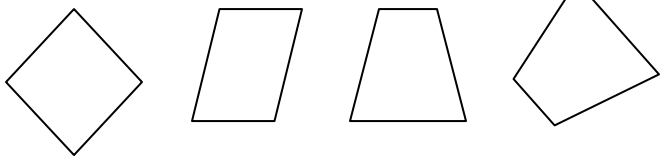
5. Can you show me which shape is congruent with Figure A? Remember, there could be more than one answer here.

Figure A

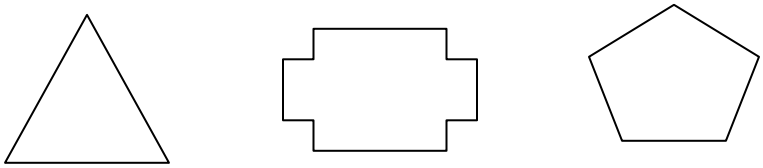


Shapes



6. Can you identify the congruent angles on each right angled triangle?

7. How many sides does this shape have? How many angles? How many vertices?



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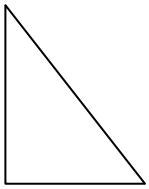
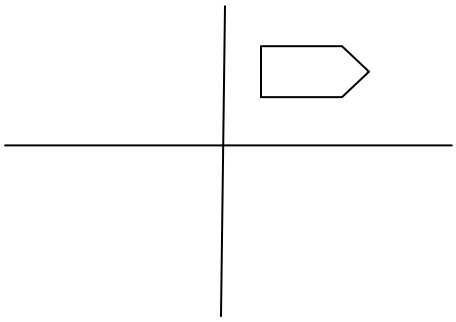
Student Worksheet at Station Two

1. Can you show me a turn, flip and slide of this shape?

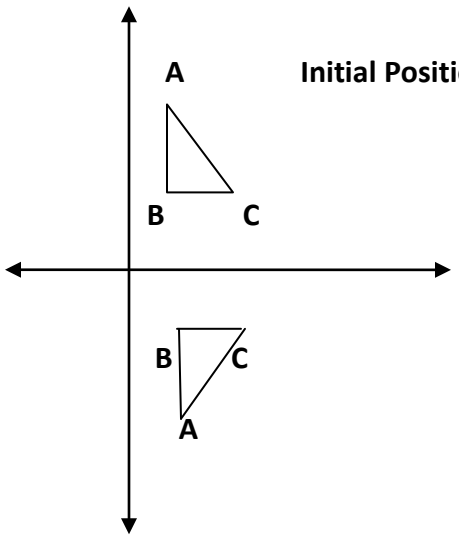


Correct Answer:

2. Can you make a rotation of the pentagon on the Geo board using the elastics?



3. The Triangle in the Initial Position" has moved. What type of movement is it in now?



Has the triangle moved into a

a) Slide

b) Turn

c) Reflection

d) Flip

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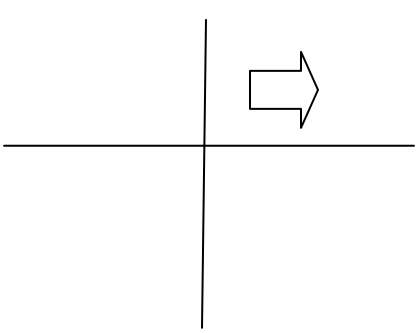
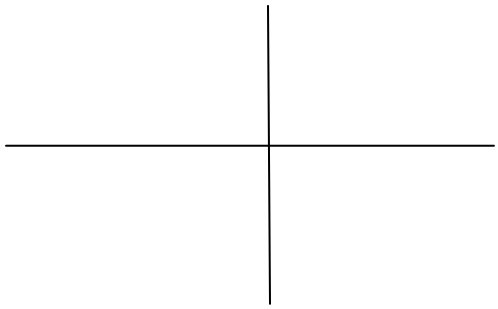
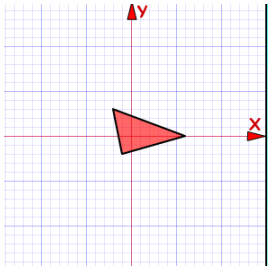
4. Can you draw this shape again in a 180 degree angle to the right?





5. Can you draw a translation for the triangle on the grid? What did you have to remember when drawing the translation? Could you draw the translation 3 unit up?



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Scoring Chart for Van Hiele's Geometric Levels of Thinking

Question	Level 0	Level 1	Level 2	Notes/Comments
Station One				
1				
2				
3				
4				
5				
6				
7				
Station Two				
1				
2				
3				
4				
5				

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The van Hiele model of geometric understanding

Table 1 The van Hiele model of geometric understanding describes a progression that is independent of age or grade level.

Level	Name	Description	Example	Teacher Activity
0	Visualization	See geometric shapes as a whole; do not focus on their particular attributes.	A student would identify a square but would be unable to articulate that it has four congruent sides with right angles.	Reinforce this level by encouraging students to group shapes according to their similarities.
1	Analysis	Recognize that each shape has different properties; identify the shape by that property.	A student is able to identify that a parallelogram has two pairs of parallel sides, and that if a quadrilateral has two pairs of parallel sides it is identified as a parallelogram.	Play the game "guess my rule," in which shapes that "fit" the rule are placed inside the circle and those that do not are outside the circle (see Russell and Economopoulos 2008).
2	Informal deduction	See the interrelationships between figures.	Given the definition of a rectangle as a quadrilateral with right angles, a student could identify a square as a rectangle.	Create hierarchies (i.e., organizational charts of the relationships) or Venn diagrams of quadrilaterals to show how the attributes of one shape imply or are related to the attributes of others.
3	Formal deduction	Construct proofs rather than just memorize them; see the possibility of developing a proof in more than one way.	Given three properties about a quadrilateral, a student could logically deduce which statement implies which about the quadrilateral (see **fig. 1**).	Provide situations in which students could use a variety of different angles depending on what was given (e.g., alternate interior or corresponding angles being congruent, or same-side interior angles being supplementary).
4	Rigor	Learn that geometry needs to be understood in the abstract; see the "construction" of geometric systems.	Students should understand that other geometries exist and that what is important is the structure of axioms, postulates, and theorems.	Study non-Euclidean geometries such as Taxi Cab geometry (Krause 1987).

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Self Directed Learning Assessment Rubrics

	< 50	50 – 59 %	60 – 69 %	70 – 79 %	80 – 100 %	Mark
Rationale *[worth 10 marks]*	The rationale is missing relevant and detailed information and as well as citations	The rationale includes unclear information with vague details. The citations are minimal and incorrect	The rationale includes some relevant information but is missing important details and correct citations	The rationale includes sufficient relevant and detailed information with citations	The rationale includes a variety of relevant and detailed information with proper citations	
Quality of Writing [Worth 5 marks]	The assignment is extremely unclear with no clarity in the presentation of ideas	The assignment is disorganized and lacks cohesion in ideas	The assignment lacks clear presentation of ideas	The assignment is well written with considerable degree of clarity	The assignment is articulate and written with high degree of clarity	
	The assignment Is filled with spelling and grammatical errors	The assignment has several spelling and grammatical errors	The assignment has some spelling and grammatical errors	The assignment has a few spelling and grammatical errors	The assignment is free of spelling and grammatical errors	
Organization of research sources *[Worth 5 marks]*	References are not listed	References are disorganized and difficult to navigate	References are show some organization	References are organized and easy to access	References are very organized and easy to access	
Assessment Script *[Worth 10 marks]*	There is no evidence of the Van Hiele theory in the script	There is limited evidence of the Van Hiele theory in the script	There is some evidence of the Van Hiele theory in the script	There is evidence of the Van Hiele theory in the script	There is clear evidence of the Van Hiele theory in the script	
	The script is disorganized , vague and impossible to follow	The script is poorly organized, with minimal details, and is difficult to follow	The script is somewhat organized with a few details, and complicated to follow	The script is considerably organized, sufficiently detailed and easy to follow	The script is highly organized, extremely detailed and very easy to follow	
Self- Created Assessment *[Worth 5 marks]*	This assessment rubric is disorganized and does not include any of the necessary criteria	This assessment rubric is poorly organized and includes only one or two of the necessary criteria	This assessment rubric is somewhat organized, and includes some of the necessary criteria	This assessment rubric is considerably organized, thorough and includes most of the necessary criteria	This assessment rubric is highly organized, thorough and includes all the necessary criteria	

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Assessment Rubric for Activity Stations

	< 50	50 – 59%	60 – 69 %	70 – 79%	80 – 100 %	Mark
Assessment Questions *[Worth 10 marks]*	The questions are not prepared in any logical order of geometrical concepts	The questions follow a poor order of geometrical concepts	The questions are prepared in a somewhat logical order of geometrical concepts	The questions are prepared in a considerably logical order of geometrical concepts	All The questions are prepared in a highly logical order of geometrical concepts	
	None of the questions are reflective of the Van Hiele's model of thinking	Only one or two questions are reflective of the Van Hiele's model of thinking	Some of the questions are reflective of the Van Hiele's model of thinking	Most of the questions are reflective of the Van Hiele's model of thinking	All the questions are highly reflective of the Van Hiele's model of thinking	
Use of Manipulatives *[Worth 8 marks]*	There are no useful manipulatives incorporated in the assessment	Only one useful manipulative is incorporated in the assessment	A few useful manipulatives are incorporated in the assessment	Some useful manipulatives are incorporated in the assessment	A variety of useful manipulatives are incorporated in the assessment	
Modifications /Extensions *[Worth 6 marks*]	The levels of difficulty for none of the questions are modified/extended based on the anticipated student responses	The levels of difficulty for one or two of the questions are modified/extended based on the anticipated student responses	The levels of difficulty for some of the questions are modified/extended based on the anticipated student responses	The levels of difficulty for most of the questions are modified/extended based on the anticipated student responses	The levels of difficulty for all the questions are modified/extended based on the anticipated student responses	
Student assessment sheet *[Worth 6 marks]*	The assessment sheet is disorganized and not coherent with the Van Hiele model	The assessment sheet for the students is poorly organized and minimally coherent with the Van Hiele model	The assessment sheet for the students is somewhat organized but is minimally coeherent with the Van Hiele model	The assessment sheet for the students is considerably organized and coherent with the Van Hiele model	The assessment sheet for the students is highly organized and coherent with the Van Hiele model	

Please write your comments below.

Total: ____/ 65

Cheers:

Next Steps:

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