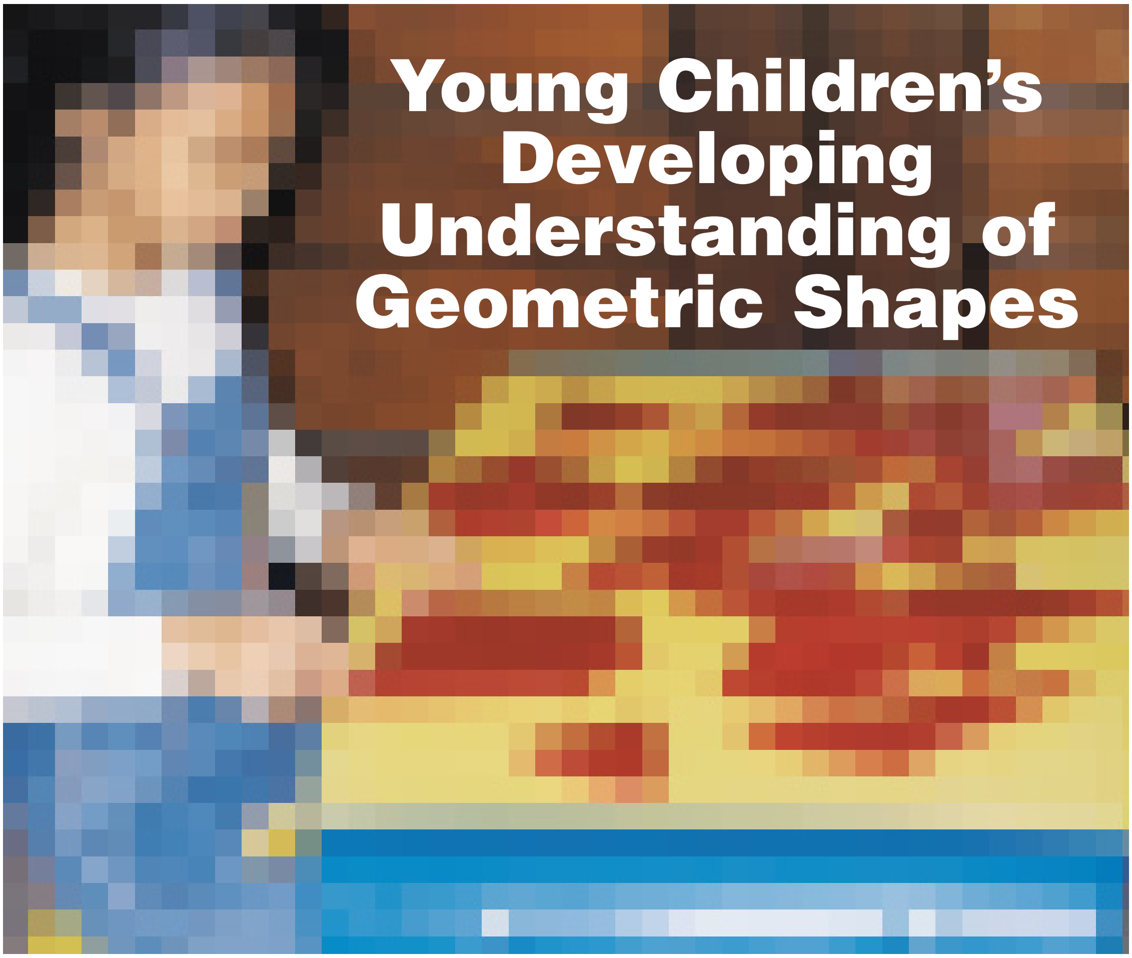

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Young Children's Developing Understanding of Geometric Shapes

Mary Anne Hannibal

How can we improve geometry instruction at the preschool and primary levels? To answer that question, I conducted research to analyze young children's understanding of the geometric concepts of triangle and rectangle and to determine patterns in the development of this understanding from ages 3 through 6. The research suggests that early childhood educators need to rethink the way that basic shapes are introduced to young children. Since a basic understanding of shapes is essential to a future study of geometry, teachers need to focus on how best to help children develop that initial understanding of shape categories. After a brief explanation of the research, specific ways to present developmentally appropriate activities designed to enhance children's understanding of basic shapes are discussed.

Related Research

Research focusing on children's concepts of space and geometric shapes began in the 1950s with psychologists' initial observations of developmental levels of geometric understanding (Piaget and Inhelder 1956, 1967). Since then, several studies have either verified (Laurendau and Pinard 1970; Liben 1978) or contradicted (Darke 1982; Dodwell 1963; Fisher 1965; Geeslin and Shar 1979; Stevens 1988) some or all of the original hypotheses. Peel's research (1959) both supported and contradicted some of Piaget's findings.

Another body of research has focused on children's reasoning about the geometric concepts that they have formed. Five sequential levels of geometric reasoning have been hypothesized by Pierre van Hiele and Dina van Hiele-Geldof (1959/1985). At the first level, the "visual level," the van Hieles propose that the child looks at a shape as a whole and not as a sum of its parts. They theorize that at this stage, the child does not attend to the properties of the shape but rather to whether it "looks like" a prototype. Therefore, at this level, an elongated triangle may not be recognized as a triangle because it is

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"too pointy" when compared with the child's mental prototype. Clements and Battista (1992) suggest that young children differentiate shapes by using a combination of a visual prototype and an unsophisticated understanding of property. They proposed that the van Hieles' visual level be redefined as a "syncretic level" of geometric understanding.

To date, the research on geometry has yet to establish a consistent pattern of development on which to base instructional programs. "Research is needed to identify the specific, original intuitions and ideas that develop and the order in which they develop" (Clements and Battista 1992).

Summary of the Study

A child's introduction to geometric shapes begins in infancy with mobiles, books, blocks, puzzles, sorting toys, and segments on various television programs. Without direct instruction, young children form an understanding of what defines a circle, triangle, and rectangle by observing and manipulating these basic shapes and identifying them by name. When children enter preschool or kindergarten, what understanding do they have of these geometric concepts, and how does this understanding develop as they mature and receive additional instruction?

The study of shapes is included in the curriculum in nearly every early childhood program. Teachers need to uncover and use the initial knowledge of shapes that children have when they enter the classroom. Only with an understanding of the young child's concept and perception of shapes can we develop a meaningful and age-appropriate geometry program.

Data were gathered by observing and interviewing twenty-four children from ages 3 through 6 as they manipulated and categorized forms as being members or nonmembers of shape categories. In two pilot studies, children as young as 3 had no difficulty identifying a circle and could even distinguish a circle from an oval. However, young children notice the nonintegral attributes of size, orientation, aspect ratio (i.e., the ratio of side lengths), and symmetry when deciding how various triangles and rectangles should be categorized. They tended to refer to how a form differed in relation to the "real" or "perfect" triangle—an equilateral triangle with the point at the top—or rectangle—similar in aspect ratio to a door. These perfect shapes are referred to as the best example or the prototypical triangle or rectangle that is most frequently presented in shape books, posters, puzzles, and toys. Therefore, for example, children would frequently not recognize a scalene triangle as a triangle because "it is too crooked."

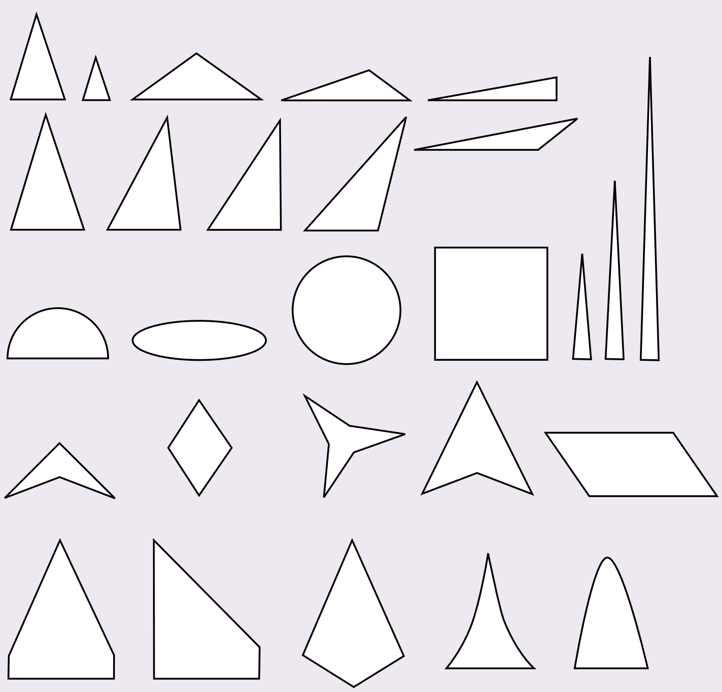
Because of these findings, specific manipulative forms were designed for this research. They included triangles and rectangles with measured variations from the prototypical shape in size, aspect ratio, or skewness (i.e., tilt) and forms that were nonrectangles and nontriangles (see **fig. 1** and **fig. 2**). These forms were presented to the children once a week for four weeks in a series of ten categorization tasks with variations in task design: a quick sort of forms without researcher probes, forms presented individually with the researcher's asking questions about categorization decisions ("Why isn't that one a triangle?" "How did you know that that one is a rectangle?"), and the presentation of forms in a hoop to test whether orientation is an important attribute to young children.

The same sorting tasks were presented a week apart to check for the constancy of children's decisions. Also, the stimuli varied in two ways. All tasks involved sorting rectangles from nonrectangles or sorting triangles from nontriangles, but on two sorts, manipulatives of other shape categories were included to study the effect of stimuli presented on categorization decisions. Six children in each of four age groups (three-, four-, five-, and six-year-olds) were interviewed, with each age group divided equally by gender.

For each sort, the percent of correct decisions that each child made, what forms the child accepted as category members, and how the child's decisions changed among similar sorting tasks

FIGURE 1

Children were given many shapes to analyze for various attributes.



were analyzed. Also studied was the acceptance rate of each form as a category member to determine the effect of aspect ratio, size, and skewness on the child's decision for inclusion.

Research Findings

A thorough study of the data highlighted several readily discernible trends in the children's developing understanding of shape concepts. Although children in each age group varied widely in their understanding of these shape concepts, the following findings by age group persisted across both triangle- and rectangle-sorting tasks:

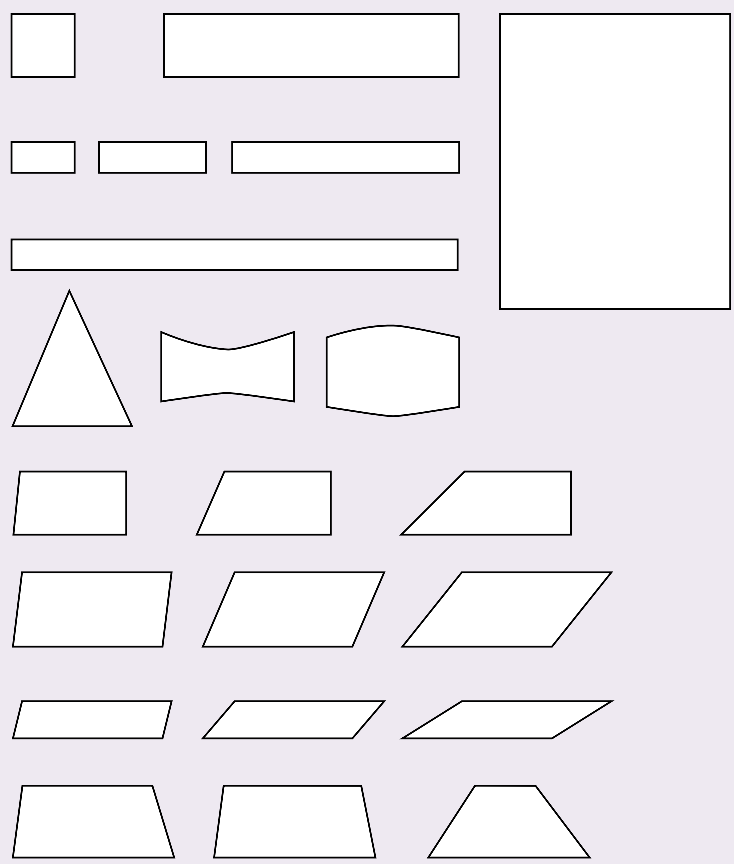
- *Finding 1:* Children's categorization decisions were influenced by the stimuli presented. For example, when a group of forms included a circle, a square, a scalene triangle, and an isosceles triangle, the children tended to accept the scalene triangle as a triangle more frequently than when the circle and square were removed and they had to choose triangles from a variety of "pointy" forms. When all pointy forms were presented and the children were asked to find the triangles, they had to rely on a more elaborate definition than "pointy" to determine which forms were triangles. Younger children begin to draw on self-determined triangle-defining criteria, such as having a point in the middle of the top, two sides the same, and three "sharp" points and being flat on the bottom.

- *Finding 2:* The constancy of categorization decisions increased from age 4 to 6. Whereas four- and five-year-old children tended to change their decisions when presented with the same sorting task a week later, six-year-olds' decisions were more consistent over the two tasks. This result leads one to believe that by age 6, children's ideas about the criteria that define a shape are becoming more fixed.

- *Finding 3:* The children tended to make more correct categorization decisions on the triangle-sorting task when they were asked to explain the reason for their decision about a form. For example, a child who was asked to justify his categorization decisions after counting the points on each form would not accept a pentagon as being a triangle. However, without researcher probes, the child would accept the pentagon as being a triangle because it had a point in the middle of the top. We can attribute this answer to the child's falling back on his more familiar, spontaneous concepts when left alone.

FIGURE 2

More shapes were analyzed.



- *Finding 4:* Children are defining the limitations for category membership in the triangle and rectangle categories from age 3 through 6. They generally begin by noticing both integral and nonintegral attributes. The fact that no six-year-old children in this study were able to correctly categorize all the forms in either shape category indicates that their concepts, although stabilized, are not completely accurate.

Implications for Teachers

This research suggests several ways that the early childhood educator can promote young children's developing understanding of geometric concepts. It is important for teachers of three-, four-, and five-year-old children to realize that at this span of ages, children are defining the limitations for membership in the triangle and rectangle categories and developing their concepts about shapes. The children at these ages are relying on both the integral and nonintegral attributes and frequently putting more weight on the nonintegral, for example, "It has three sides, but it is too long to be a triangle."

Teachers need to move beyond having children just label shapes to having them understand what defines a shape category. They must help children distinguish the integral attributes of number of sides and number and size of angles from the nonintegral attributes of size, aspect ratio, orientation, and, in the case of triangles, symmetry by presenting a wide variety of triangles and rectangles rather than only the prototypical forms. Also, since it was observed that several children did not understand the difference between "points" and "sides," teachers need to clarify these terms before offering the mathematical definition of a shape.

As children manipulate and sort shapes, teachers should ask them to verbalize their reasoning about shape categories. This verbalization not only provides valuable information to the teacher about the child's understanding of a concept, it also helps the child incorporate more scientific understandings into his or her knowledge base.

Early childhood teachers introduce four categories of shapes: circles, squares, triangles, and rectangles. The idea that a square is not a rectangle is firmly rooted by age 5, as no five- or six-year-old in this study would accept the square as being a rectangle. I suggest that it is time to rethink our presentation of squares as a set unto themselves, so that children develop a more accurate understanding of the basic geometric concept: a square is a rectangle with four congruent sides.

When presenting geometric concepts to even the youngest children, teachers need to be intellectually honest and present mathematically correct definitions of shapes and shape sets. The following guidelines for introducing triangles and rectangles to young children are based on the mathematical definitions of these shapes.

Introducing triangles

Do say that triangles have three sides, or line segments, and three points, or corners, with all sides straight and all sides connected. Point out that triangles may vary in orientation, size, symmetry, and "pointedness."

Do not say that triangles—

- have two points at the bottom and one at the top.
- have a point in the middle.
- have a flat bottom.
- are pointy.
- are like the open triangle used in music class or a cone-shaped clown hat.
- can be made from any three line segments.

Introducing rectangles

Do say that rectangles have four sides with opposite sides congruent and have four right angles. Show students how to use a corner of a piece of paper as a "right-angle checker." Do not limit the definition to four sides only, as this statement will lead children to accept all quadrilaterals as being rectangles. All sides are connected and straight and rectangles are symmetrical. Cutting rectangles in half and comparing the pieces or making rectangles by combining identical right triangles helps children understand symmetry. The corners of rectangles never vary, although rectangles can vary in size and orientation.

Do not say that rectangles—

- are long.
- have two long sides and two short sides.
- are like any three-dimensional shape, such as a shoe box.

Suggestions for Classroom Activities

The following suggestions for developmentally appropriate classroom activities will help children learn the correct concepts of shapes and lay the groundwork for their success in geometry:

- Look around your classroom at puzzles, books, posters, and manipulatives. Are equilateral triangles always presented as the example, or are scalene, obtuse, isosceles, and right triangles also presented? Do rectangles vary in size, orientation, and aspect ratio? If not, think about varying these shapes for children to manipulate and sort.
- Integrate the study of shapes into all areas of the curriculum.
- Read and discuss books about shapes, or point out shapes in storybook illustrations.
- Place shapes in a "feely bag," and have children choose by touch the shape that matches the one that you are holding.
- Find shapes around the room.
- Display projects at the art table. Have children make shapes from other shapes, make shapes from play dough, talk about the shapes in children's pictures, make pictures using cutout shapes, sponge paint with shapes, and use shape templates.
- Make "shape bingo" cards displaying a variety of shapes.
- Initiate gross-motor activities, such as having children walk around large shapes on the floor and work as a group to make shapes with their bodies.
- Incorporate shapes into music by singing songs about shapes and discussing the shapes of rhythm instruments.
- Make shape sandwiches as a cooking activity.

Finally, be certain that both boys and girls are working and playing with shapes. These activities should not be left to self-selection. Boys tend to gravitate to the Legos and the block corner, where they spend considerable time manipulating shapes. In this study, boys consistently made more correct categorization decisions than the girls, with the gap widening as the children got older. Therefore, teachers need to select shape-manipulation activities that are attractive to the girls, too. For example, some teachers initiate a "girls only day" in the block area to spark girls' interest in building with blocks.

Conclusion

This study analyzed young children's understanding of the geometric concepts of triangles and rectangles, and defined patterns in the development of this understanding from ages 3 through 6. An understanding of how young children perceive geometric concepts and of how these perceptions develop as the child both matures and receives systematic instruction is imperative if teachers are to improve early childhood geometry instruction. Early childhood educators should define the essential attributes of shapes for children and demonstrate the irrelevance of orientation, aspect ratio, size, and symmetry. Developmentally appropriate activities should present geometric shapes to young children in an intellectually honest manner, paying close attention to the mathematically correct definitions of shapes and shape sets.

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