

A VIRTUAL SPIN on the Teaching of Probability

The study of probability is part of one of the five major mathematics content strands as defined by *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics 2000). Traditionally, teaching the concepts of probability has required students to use concrete manipulatives, such as coins and number cubes, in order to perform various probability experiments. With the continued emphasis on integrating technology into the mathematics classroom, the teaching of probability has acquired a “virtual spin”—students now use various virtual manipulatives to perform probability experiments.

Research on the effective use of technology in the mathematics classroom has indicated certain advantages of using virtual manipulatives rather than concrete manipulatives. Advantages include a linkage from the specific to the general, the possibility of posing a wider variety of problem types, the establishment of a strong foundation for problem solving, an increased focus of attention and motivation, and an encouragement of accurate student explanations for mathematics strands (Clements and McMillen 1996).

Exploring Probability by Using Concrete Manipulatives

Before the introduction of virtual simulations, the study of probability typically began with students’ physically flipping coins to collect experimental data, starting with flipping one coin and progressing to flipping three or more coins at a time. The students constructed tree diagrams to create a list of all possible outcomes of each experiment. The collected data and the constructed sample space (the set of all possible outcomes) were used to compare and contrast experimental and theoretical probability. Finally, the students used the number of elements in a sample space for a specified experiment to decide on a sufficient number of times to perform the experiment.

A second concrete manipulative we used to teach probability was assorted spinners that included various combinations of colors and numbers. Some of the spinners had equal-sized regions for each color, while others had multiple regions of certain colors. The students themselves also built spinners to satisfy specified criteria.

A third concrete manipulative we used to teach probability was number cubes. Students would initially roll a single six-sided number cube and then roll a pair of cubes in order to analyze a compound experiment containing independent events (results where the outcome of one event is *not* dependent on the outcome or occurrence of another event). Later in the unit students would combine two activities—that is, they would both flip coins and roll number cubes. In each case, we encouraged students to visualize the sample space of possible outcomes by drawing a tree diagram.

The probability unit may have ended with an

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experiment such as the Matching Sock problem, which would have been stated this way:

A drawer contains 4 pairs of purple socks, 1 pair of green socks, and 1 pair of yellow socks. The socks are not folded together in matching pairs but are mixed together in the drawer. What is the probability of pulling 2 socks from the drawer and having a matching pair?

We encouraged students to analyze this probability problem by constructing a tree diagram of possible pairings, a tool that provided a visual way of counting the possible matching pairs of socks.

Exploring Probability by Using Virtual Manipulatives

In the summer of 2005, we taught all the same topics in the probability unit by using virtual manipulatives available for free on the Internet. Virtual manipulatives have been defined as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer et al. 2002, p. 373). The National Library of Virtual Manipulatives (NLVM) for Interactive Mathematics, found at nlvm.usu.edu/en/nav/vlibrary.html, provides an extensive resource of more than seventy-five virtual manipulatives developed at Utah State University. For students who do not have adequate supplies of concrete manipulatives available, this Web site provides a free alternative when computers with access to the Internet are available (Hodge 2003). This Web site contains an index of various activities from each of the five Content Standards as outlined by the National Council of Teachers of Mathematics in *Principles and Standards for School Mathematics* (NCTM 2000). The virtual manipulatives are broken down into groups by levels: preK–grade 2, grades 3–5, grades 6–8, and grades 9–12. **Figure 1** shows the library entry page to the NLVM.

The “virtual spin” on the probability unit began with coin tossing, as had been the traditional approach, but now the students used the virtual coin toss from the NLVM. One advantage that we noticed immediately was the amount of time saved by having the computer toss the coins. When using actual coins, the students typically never flipped a coin more than twenty times in a single experiment, whereas the computer could toss the coin hundreds of times in the same amount of time. The students



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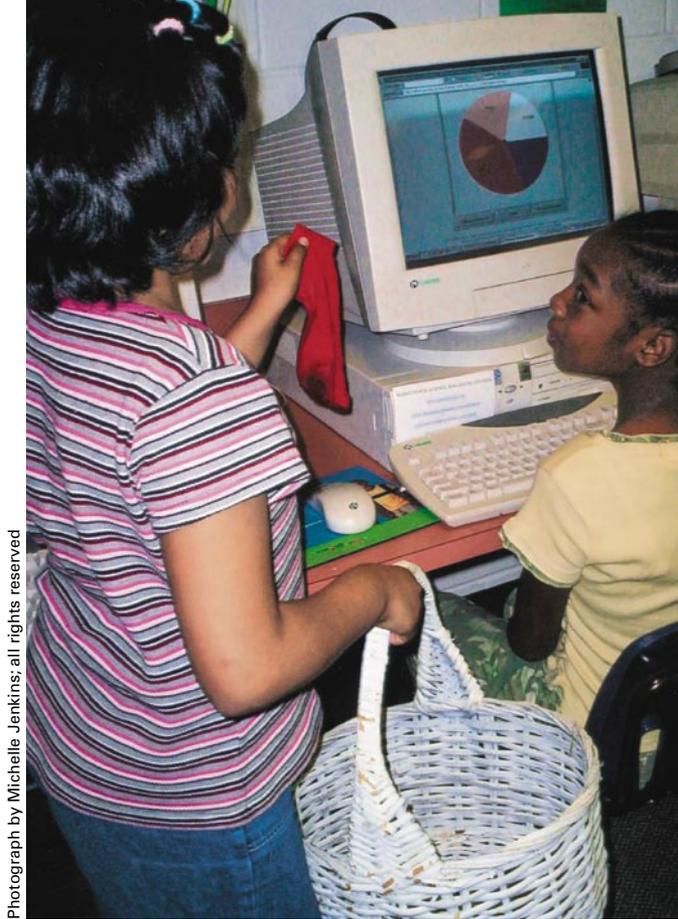
Figure 1

Library entry page of the National Library of Virtual Manipulatives (NLVM) Web site: nlvm.usu.edu

Index	Pre-K–2	3–5	6–8	9–12
Number & Operations				
Algebra				
Geometry				
Measurement				
Data Analysis & Probability				

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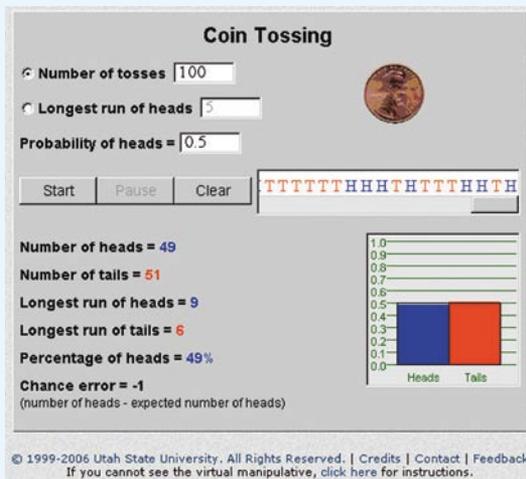
Source: National Library of Virtual Manipulatives, nlvm.usu.edu (Copyright © Utah State University)



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Figure 2

Coin tossing on the NLVM Web site



Source: National Library of Virtual Manipulatives, nlvm.usu.edu (Copyright © Utah State University)

quickly collected data for 100 coin flips and analyzed the experimental outcomes versus the theoretical probability of what was expected to happen. As the computer flipped the coin, it also displayed the data on a bar graph, a quick way to visually analyze the results. The students watched the heights of the bars fluctuate as the frequencies of the outcomes changed with every flip. For one particular experiment, the bar graph indicated a higher frequency of

tails (see **fig. 2**). Results varied from one computer to the next, and the students noticed different bar graphs for each of their experiments.

We adapted the experiment to include two or three coins by grouping students and having each group collect data for an individual coin. We continued to teach the students how to draw a tree diagram of the compound experiment in order to construct a sample space of all possible outcomes. In their small groups the students, taking into account the number of outcomes in each sample space, discussed how many times an experiment should be performed in order for all outcomes to have a chance to show up an equal number of times. For example, because flipping three coins could result in eight different outcomes, the students determined that the number of flips in the experiment needed to be some multiple of eight. Again, they were able to perform numerous experiments in a shorter period of time because they did not have to physically flip the coins. As a result, we were able to focus more on the data collected from the experiment than the physical ability to flip the coin.

The next experiment with virtual manipulatives included the spinner. The lesson began with the students using the default spinner, which contained five equal-sized regions of different colors (see **fig. 3**). Again, the students were able to include hundreds of spins in their experiment, and they analyzed the results of the spins by means of the bar graph that the computer constructed as the experiment was being performed (see **fig. 4**). As before, each group compared its bar graphs to those of other groups, and the students noticed the differences in their results.

We then asked the students to change the colored regions so that the sections were not all of equal sizes. The NLVM allowed users to specify how many regions of each color should appear on the spinner. The students made predictions about expected outcomes on the basis of which color made up the largest area on the spinner and then conducted experiments to include hundreds of outcomes each time. Again, our class time was focused more on the teaching of probability and not on the physical manipulation of the spinner.

We extended the lesson by using the virtual spinner to conduct experiments with both a single number cube and a pair of cubes. (The NLVM site does not include number cubes, but they can be found on other virtual manipulative Web sites.) To turn the spinner into a single number cube, the

students relabeled the spinner's regions (see **fig. 5**) to include numbers rather than the names of the colors. Each equal-sized region was specified as a different color, but the focus was on the numerical outcomes, which represented the outcomes of rolling a single number cube. As before, the students conducted the experiment hundreds of times and visually analyzed the results reflected in the bar graph that the computer constructed as the number cube was being rolled.

We further extended the lesson with an experiment to include a pair of number cubes. We grouped the students together and had them pair their outcomes on each single number cube. We also paired the students for a different compound experiment in which one of them rolled a single number cube while the other one flipped a coin. Again, drawing a tree diagram of each of these compound experiments helped the students understand all possible outcomes in the sample space, and, on the basis of the number of different outcomes, they determined how many times an experiment needed to be performed.

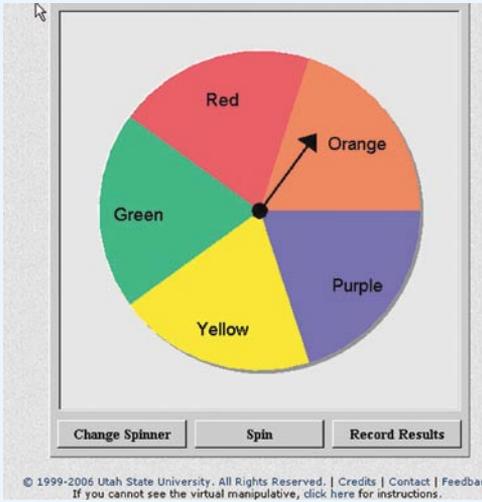
One of the neatest experiments the students conducted by using virtual manipulatives focused on solving the Matching Sock problem stated earlier. Using the virtual spinner, the students allocated two regions on the spinner as purple to represent the 2 purple socks in the drawer and allocated four regions on the spinner as red to represent the 4 red socks. In the same manner, the students allocated two regions of the spinner as pink and two regions of the spinner as white. The computer gave the students a good visual representation of the dominance of the color red among the socks in the drawer (see **fig. 6**), and they quickly concluded that the matching pair was likely to be red. The students began by spinning one time and recording the results.

After this first spin and after the first sock had been virtually pulled out of the drawer, we asked the students whether the drawer would still look the same—that is, whether the proportion of colors in the drawer would still be the same. The students realized that they needed to change the color regions on their spinner to include one less region for the color of the sock first pulled from the drawer.

Because the students had to change the color regions on the spinner, our teaching focused on the meaning of this experiment's dependent events (in dependent events the outcome of one event is dependent on the outcome or occurrence of another event, and the possible outcomes are related or conditional)—here, the proportion of colors in

Figure 3

A default spinner on the NLVM Web site



Source: National Library of Virtual Manipulatives, nlv.m.usu.edu (Copyright © Utah State University)

Figure 4

Spinner results in a bar graph format

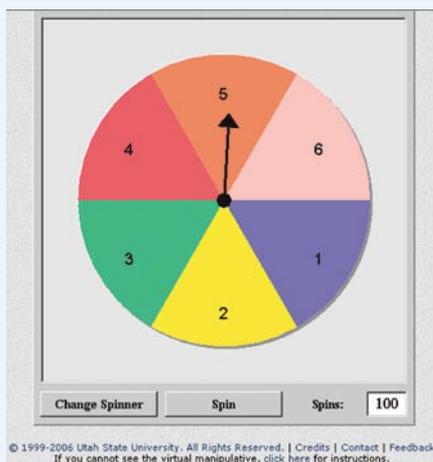


Source: National Library of Virtual Manipulatives, nlv.m.usu.edu (Copyright © Utah State University)

the drawer is dependent on which color of sock is pulled out of the drawer. This contrasted with our previous experiments, which involved independent events (that is, the outcome of one coin flip is *not* dependent on the outcome of another coin flip). Once the students changed the color allocation on their spinners, they spun a second time to determine the color of sock that would next be pulled from the drawer. After all the groups had virtually pulled two

Figure 5

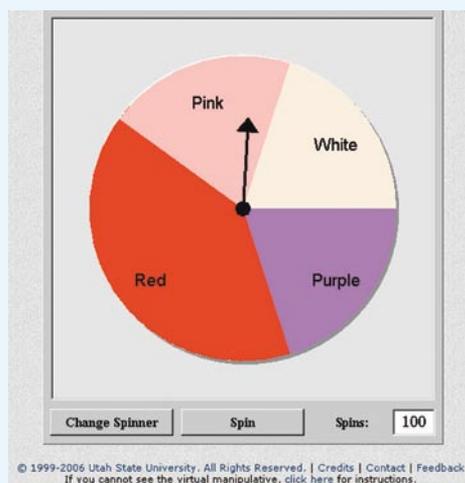
Spinner for the Number Cube problem on the NLVM Web site



Source: National Library of Virtual Manipulatives, nlvvm.usu.edu (Copyright © Utah State University)

Figure 6

Spinner for the Matching Sock problem on the NLVM Web site



Source: National Library of Virtual Manipulatives, nlvvm.usu.edu (Copyright © Utah State University)

socks from the drawer, they analyzed the class's collective results and quickly noticed two things: a larger number of mismatched pairs remained in the drawer, and, of the pairs of socks of matching colors, purple pairs dominated.

We extended the lesson even further by having the students draw the tree diagram once again to look at all possible outcomes. Then they used the

spinners again to pull socks from the drawer. The principle of dependent events was clear: The color of the first sock drawn affected the possible outcome of the second sock drawn.

Reflections

Our study of probability using virtual rather than concrete manipulatives allowed us to spend more time on teaching and allowed the students more time to build sample spaces. In addition, the students were able to view a larger number of outcomes for each experiment. And because the students were able to perform experiments hundreds of times more than when they physically had to manipulate the concrete objects, they could explore the law of large numbers: The larger the sample, the more likely the result is a good predictor for the whole population. Throughout our study of probability, the students learned how to construct their own sample spaces and made predictions of the theoretical probability of certain outcomes. Then, they were able to compare their predictions with the experimental results that actually occurred. Because the students had to physically make changes to the spinner during an experiment involving dependent events, they were able to quickly recognize the difference between dependent events and independent events. The “virtual spin” on this teaching of probability provided more opportunities for successful learning for the students.

Bibliography

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Additional Resources

Readers looking for lessons and activities that make use of virtual manipulatives will find an abundance of resources at <http://illuminations.nctm.org/> and nctm.org/onmath.—Ed. ▲