Balancing Act: The Truth behind the Equals Sign

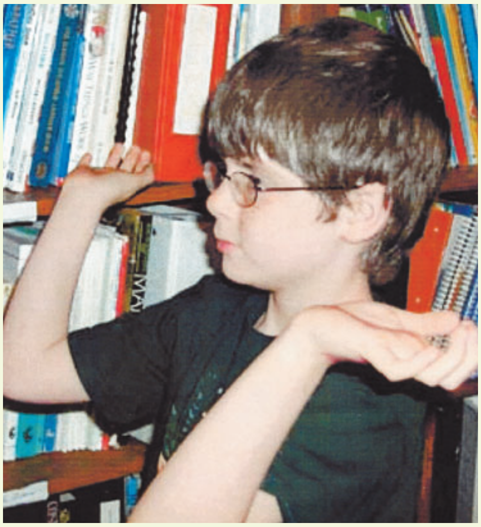
I walked into the classroom, wrote " = " on the board, and asked the third graders, "What does this mean?" You can probably anticipate the response—the students all thought it meant "The answer is." Only after some gentle nudging did the students agree that it could also mean "is the same as."

Thinking of the equals sign operationally is a common interpretation of many elementary school students (Kilpatrick, Swafford, and Findell 2001; NCTM 2000). Falkner, Levi, and Carpenter (1999) state, "Children in the elementary grades generally think that the equals sign means that they should carry out the calculation that precedes it and that the number after the equals sign is the answer to the calculation. Elementary school children generally do not see the equals sign as a symbol that expresses the relationship 'is the same as' " (p. 233). The equals sign is a symbol that indicates that a state of equality exists and that the two values on either side of the equals sign are the same. It does not mean that the answer is coming or that the answer is on the other side of the sign.

An understanding of the concept of equality is vital to successful algebraic thinking and is one of the big ideas of algebra about which students should reason. The concept of balance, or equivalence, is the basis for the comprehension of equations and inequalities (Greenes and Findell 1999). Exposing students to this important algebraic concept in the lower grades is essential to develop an understanding of equality (NCTM 2000). Instead of waiting to introduce the concept during the middle school years, teachers should help students in elementary school come to recognize the equals sign as a symbol that represents equivalence and balance.

Figure 1

A student simulates a balanced seesaw.



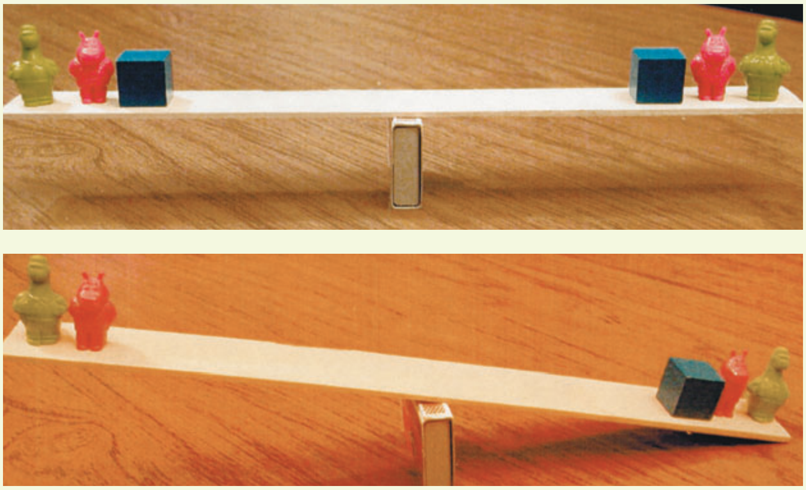
Photograph by Rebecca L. Mann; all rights reserved

By Rebecca L. Mann

Rebecca Mann, rebecca.mann@uconn.edu, is a doctoral candidate at the University of Connecticut in Educational Psychology/Gifted and Talented and Special Education. She is a former gifted-and-talented resource teacher and elementary classroom teacher. She has a special interest in concepts-based mathematics instruction as well as in gifted and learning-disabled students.

Figure 2

Students' seesaws



To help my students transition from "The answer is" to the "is the same as" mode of thinking, I initiated a discussion about seesaws:

Me. What happens when you go on a seesaw?

Sara. You go up and down and if the other person is big, like your dad, you stay up in the air.

Me. What do you think Sara meant when she said if the other person is big, you stay up in the air?

Matthew. She means that if the other person is heavier, then you sometimes only go up and can't come down no matter how hard you try.

Steven. Yeah, but if I go on with my little brother, I go down and I can't stay up in the air. Even if I try, I keep coming back down.

Me. Why do you suppose that happens?

Maria. It's because you weigh different. I mean, because Sara and her dad or Steven and his brother don't weigh the same. See, when Arcelia *(her twin sister)* and I go on the seesaw, we can balance it. That's because we are the same size.

Me. So it is easier to balance the seesaw if you are both the same size? *(The class agrees.)*

The students then became seesaws. With elbows bent and palms at shoulder level facing the ceiling, each student simulated a balanced seesaw (see **fig. 1**). I told the class that I had an imaginary basket of oranges, stressing that each big, fat, juicy orange weighed exactly the same as every other orange. I also told them about my imaginary basket of apples. These poor apples were the scrawny leftovers. Each apple weighed the same as the others, but each apple weighed much less than each orange. Next, I took the students through a series of scenarios. "I just put an orange in your right hand," I said. "What happened to your seesaw?" The students leaned to the right. "Now I am coming around and putting an orange in your left hand," I continued. "Remember that the oranges weigh the same. Now what happens to your seesaw?" The students straightened up, bringing their seesaws into balance. "The oranges are still in your hands and I am going to add an apple to your left hand," I said. Without waiting for the question, the class quickly tipped to the left. "What should we do to bring you into balance again?" I asked. "Add an apple," Jamal replied. "OK, I am coming around and adding an apple to your left hand," I said.

Some of the students stood up straight, a few leaned even farther to their left, and the majority looked confused. A discussion began about the placement of the last apple. The class came to the consensus that I had made a mistake. "So what you are telling me is that if I had wanted to balance the seesaw, it makes a difference as to on which side I place the apple?" I asked. The students responded, "Yes!"

"OK!" I said. "I'll take the apple I just added to your left hand and move it to your right hand."

The students looked relieved to be back in balance. The lesson continued as I "removed" apples and oranges in different arrangements from the students' seesaws. I made a point during the activity to add one orange to each hand at the same time, to demonstrate how adding equal weight to both sides of the seesaw does not change the tilt of the seesaw.

After shaking out their tired arms, the students were asked to create a set of "Seesaw Rules." Students worked in small groups to discuss their ideas and develop a series of statements about balancing seesaws as I moved from group to group, encouraging the students to extend their thinking. When I had determined that all the essential ideas were incorporated into at least one of the lists, the groups shared their lists and came to consensus as a class on the following set of rules:

- For a seesaw to be balanced, it must have the same amount of weight on each end.
- If one end weighs more or less, the seesaw will not be balanced.
- If you have a balanced seesaw and add something to one end, it will not be balanced anymore.

- If you have a balanced seesaw and take away something from one end, it will not be balanced anymore.
- If you have a balanced seesaw and add the same amount of weight to both ends, it will still be balanced.
- If you have a balanced seesaw and take away the same amount of weight from both ends, it will still be balanced.

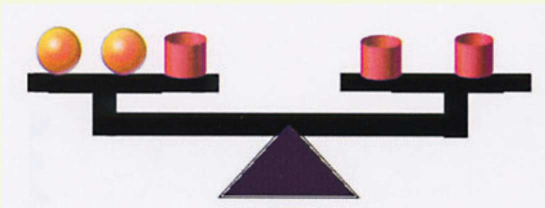
Next, I directed the students' attention to the blackboard, on which I had written the equals sign. I encouraged students to make a connection between the equals sign and their seesaws.

Me. What does this equals sign on the board have to do with your seesaws?

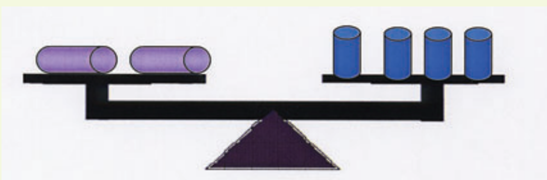
Debbie. If you have two oranges on one side and two oranges on the other side, it's the same.

Figure 3

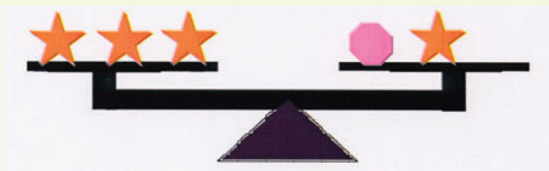
Activities designed to encourage the development of number sense



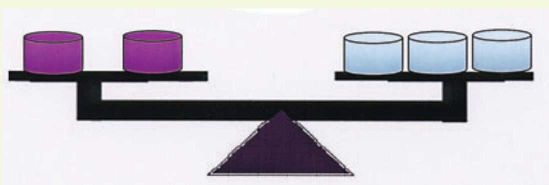
The red cylinders each weigh the same. The yellow balls weigh the same. The scale is balanced. What do you know about the weights of the balls and the cylinders?



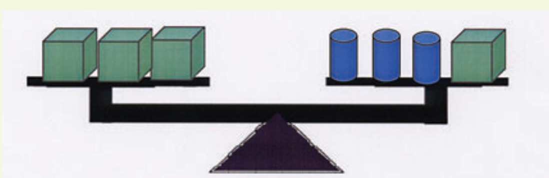
Each purple cylinder weighs the same. Each blue cylinder weighs the same. The scale is balanced. What do you know about the weights of the cylinders?



Each orange star weighs the same. The scale is balanced. What do you know about the weights of the stars and the octagon?



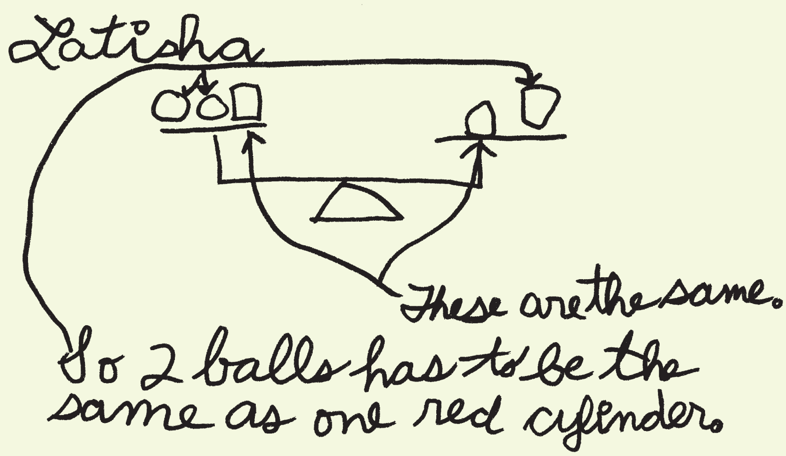
Each purple cylinder weighs the same. Each blue cylinder weighs the same. The scale is balanced. What do you know about the weights of the purple cylinders and the blue cylinders?



Each green cube weighs the same. Each blue cylinder weighs the same. The scale is balanced. What do you know about the weights of the cubes and cylinders?

Figure 4

Latisha's work on the problem



Theo. Yeah, you have to have the same thing on both sides of the seesaw and you have to have the same thing on both sides of the equals sign.

Blake. Look at our rules! If you want the seesaw to be balanced, the same weight must be on each side and if you take something away from one side, it won't be balanced. It's the same for the equals sign.

Me. Can anyone explain what Blake is talking about?

Sari. Well . . . if you have 3 oranges and 2 apples on one side of the equals sign, just like we had on the seesaw, and 3 oranges and 2 apples on the other side of the equals sign, you have the same on both sides. If you took 2 apples away from one side, it wouldn't be equal anymore; you wouldn't have the same on both sides.

Nancy. Yeah! And it wouldn't balance. See? To be equal, both sides have to balance. Cool!

The third graders were beginning to see the connections between the equals sign, equivalence, and balance.

After this initial "seesaw" lesson, I encouraged students to revisit the balance idea on a regular basis. Over the course of the school year, I placed emphasis on viewing the equals sign not as an indication that it was time to execute an operation but as an indicator of the presence of a relationship. Falkner, Levi, and Carpenter (1999) state,

> A concerted effort over an extended period of time is required to establish appropriate notions of equality. Teachers should also be concerned about children's conceptions of equality as soon as symbols for representing number operations are introduced. Otherwise, misconceptions about equality can become more firmly entrenched. (p. 233)

Additional activities to which the students were exposed that reinforced the concepts of balance and equivalence were pan-balance scales, missing-addend problems, and open-number sentences such as $5 + 6 = \triangle + 2$ or $7 - 3 = 1 + \diamond$. If students were in doubt about their solutions, they were encouraged to act out the problem on their own created seesaw (see **fig. 2**). Students also had opportunities to experiment with balance-scale activities that encourage the development of number sense; in these activities, balance is maintained without assigning numerical values to different shapes. **Figure 3** illustrates activities designed to encourage the development of number sense. After analyzing the contents of the two pans, students compare the two sides and use their deductive-reasoning skills to determine relationships between the objects, as Latisha did in **figure 4.**

The concepts of equivalence and balance are an essential first step in algebraic thinking. In preparation for manipulating equations in secondary school, students should learn how to create and maintain balance (equality) in the lower grades (Greenes et al. 2001). Thinking algebraically should not be the domain of middle school or high school students. The foundation can be, and should be, set during the primary years.

Resources for Teaching Equivalency

The resources listed below contain balance-scale activities appropriate for students in kindergarten through fifth grade.

Cuevas, Gilbert J., and Karol Yeatts. *Navigating through Algebra in Grades 3–5.* Reston, Va.: National Council of Teachers of Mathematics, 2001.

Gavin, M. Katherine, Carol R. Findell, Carole E. Greenes, and Linda Jensen Sheffield. *Awesome Math Problems for Creative Thinking*. Series of six books. Mountain View, Calif.: Creative Publications, 2000.

Greenes, Carole, and Carol Findell. *Groundworks: Algebraic Thinking*. Palo Alto, Calif.: Creative Publications, 1999.

Greenes, Carole, Mary Cavanagh, Linda Dacey, Carol Findell, and Marian Small. *Navigating through Algebra in Prekindergarten–Grade 2*. Reston, Va.: National Council of Teachers of Mathematics, 2001.

Hoogeboom, Shirley, and Judy Goodnow. *Beginning Algebra Thinking for Grades 3–4*. Alsip, Ill.: Ideal School Supply Company, 1994.

References

Falkner, Karen P., Linda Levi, and Thomas P. Carpenter. "Children's Understanding of Equality: A Foundation for Algebra." *Teaching Children Mathematics* 6 (4) (December 1999): 232–36.

Greenes, Carole, Mary Cavanagh, Linda Dacey, Carol Findell, and Marian Small. *Navigating through Algebra in Prekindergarten–Grade 2*. Reston, Va.: National Council of Teachers of Mathematics, 2001.

Greenes, Carole, and Carol Findell. "Developing Students' Algebraic Reasoning Abilities." In *Developing Mathematical Reasoning in Grades K–12*, 1999 Yearbook of the National Council of Teachers of Mathematics, edited by Lee V. Stiff. Reston, Va.: National Council of Teachers of Mathematics, 1999.

Kilpatrick, Jeremy, Jane Swafford, and Bradford Findell, eds. *Adding It Up: Helping Children Learn Mathematics.* Washington, D.C.: National Academy Press, 2001.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000. ▲