



Why Can't I See the Tree? A Study of Perspective

Developing an understanding of three-dimensional perspective is an important life skill; we all make judgments continuously about the relationships among three-dimensional objects. We use these relationships to navigate, understand, and follow directions, construct things, and make interpretations and decisions about arranging objects in our homes or offices.

Many school topics require an understanding of perspective. Social studies courses require map-reading and scale-drawing skills. Students in science classes record and process information using their understanding of structures, place and location, and other relationships. In mathematics, *Principles and Standards for School Mathematics* (NCTM 2000) states that instructional programs should enable students to use visualization, spatial reasoning, and geometric modeling to solve problems. Expectations are that in grades 3–5, all students should—

- build and draw geometric objects;
- identify and build a three-dimensional object from two-dimensional representations of that object;
- use geometric models to solve problems in other areas of mathematics, such as number and measurement; and
- recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life. (NCTM 2000, p. 164)

When teachers use manipulatives to teach arithmetic, they assume that their students possess many of the skills and understandings called for in the NCTM Standards. They use the manipulatives to help children develop images that they believe will aid in understanding how numbers and algorithms

work. Unfortunately, many young adults and children have not developed these skills and understandings. Some of my college students over the years were unable to explain where the 1000 smaller cubes were in a larger block that was part of the base-10 materials. They could not visualize that a picture of a block, which shows three faces, represents a three-dimensional block with six faces, or that a picture of a block taken from a corner view is the same block shown in a picture taken from a side view. These tasks can cause difficulties for young children who have not developed the mental schemas needed for working with and connecting two- and three-dimensional representations.

As teachers, it is imperative that we spend more time assessing and developing our students' existing spatial understanding. In an attempt to do this, I have observed numerous children and young adults, over long periods of time, engaged in specific activities designed to reveal their understanding of perspective and to develop missing skills. The assessments and activities in this article specifically address the NCTM Geometry Standards, which state that children in grades pre-K–2 should be able to—

- describe, name, and interpret relative positions in space and apply ideas about relative position;

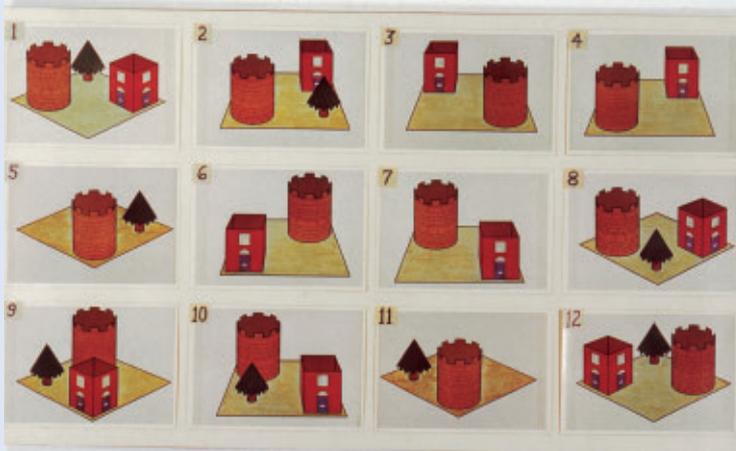
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Edited by Carol Langbort, clangbo@sfsu.edu, Department of Elementary Education, San Francisco State University, San Francisco, CA 94132. Readers are encouraged to send manuscripts appropriate for this section to "Research, Reflection, Practice," NCTM, 1906 Association Drive, Reston, VA 20191-1502.

Find the picture

The child is asked to select the picture that Nellie, the doll in **figures 3** and **4**, would see as she is moved to the seven positions, not including “A,” indicated by the letters in **figure 5**.



Find the picture

This is the scene that the child sees at the beginning of the assessment. The child is seated at position A, the same position as the camera in **figures 2, 3, and 4**.



- describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance; and
- find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.

Children also should be able to use visualization to—

- recognize and represent shapes from different perspectives; and
- recognize geometric shapes and structures in the environment and specify their location. (NCTM 2000, p. 96)

Piaget and Inhelder (1956) engaged in extensive studies of the mental development of perspective

and concluded that its development progresses through stages. The Nuffield Project (1972) devised protocols, in conjunction with Piaget, for checking children’s stages of development of perspective. I have made several changes and additions to these protocols, based on my own research (Kelly, Kelly, and Johnson 1988) and the research of Laurendeau and Pinard (1970) and Clements (1998). I have found these protocols very useful in assessing an individual’s level of functioning and in guiding me to create developmentally appropriate activities that are easy to make and easy to use. Because these activities assess the most basic understanding recommended by the NCTM Standards for grades pre-K–2, I have been surprised to find that many young adults, particularly females, have difficulty with them.

A mature understanding of perspective includes: (1) familiarity with the words and concepts that describe changing relationships among objects as one hypothetically assumes differing positions, such as *to the right, to the left, in front, and behind*; (2) the ability to look at a scene and predict what a picture of the same scene would look like from different positions around the table; and (3) the ability to look at a picture of a scene and predict where one would have to be located around the table in order to see what is pictured. Both the assessments and the teaching activities that follow perspective exercises must include tasks to assess, and later develop, each of these abilities. Young children’s vocabularies and abilities to describe situations often are not as well developed as their emerging understandings. Therefore, these tasks do not ask for oral description; instead, they ask the child to give his or her response in a variety of ways.

In order to complete the following assessment tasks, you need three simple objects, differing in size. They should be large enough to be seen from several feet away, but small enough to fit in the center of a table at which a child can be seated. The child should be able to walk around the table easily. The objects should be at about eye level when the child is seated at the table. For the initial assessment, each of the three objects should look the same on all sides. The focus at the beginning of the assessment is on relative positions between the objects, rather than the characteristics of the individual objects.

Assessment Tasks

Task 1: Find the picture

In this first task, the child is shown a set of pictures (see **fig. 1**) and asked to select the picture that the

doll would see if it were sitting at various places around the table. The doll is used to mark the selected viewing position for the child. This makes the task more concrete and easier for the child to visualize. It is important to note that only the doll is moved. Positions are indicated with letters in **figure 5**. The child remains in his or her seat and must visualize and identify the picture of the view that would be seen from the doll's position. (The views are shown in **figures 2, 3, and 4**.)

Task 2: Find the place

In this assessment task, the child is shown a picture of a scene from one of the positions around the table (see **fig. 5**) and asked where he or she would have to move in order to see the scene as pictured. During this assessment, the child remains seated at the starting position with the view pictured in **figure 2** and marked with an "A" in **figure 5**. Note that the "base" of the pictures that the child is shown (see **fig. 6**) is round, whereas the base for the scene itself is square. The reason for this is to discourage the use of sides and corners as visual cues so the emphasis is on the relative positions of the objects. Although the "find the picture" task is just the reverse of the "find the place" task, I have found that children who are successful at one task may not be successful at the other. A child may have an incomplete understanding of differing perspectives if both skills are not developed.

In this task, it is important to evaluate how the student handles the photographically reversed images, the pairs of which are the first and eighth, fourth and fifth, and ninth and tenth shown in **figure 6**. I have found in my research that these pictures help to determine the stability of students' understanding of relative positions.

Student Responses

Following are responses from three representative children that demonstrate their various levels of thinking. Katie is in grade 2 and is 7 years and 5 months old, Mary is in grade 2 and is 7 years 4 months, and Sally is in grade 3 and is 9 years 2 months. Assessments were done individually. To begin the assessment, each student was asked what she thought the pictures on the board represented (see **fig. 1**). All the students identified them as photographs of the scene on the table in front of them. Each child was then asked to select the picture that showed the scene she was looking at. (Each student was seated at the starting position, and the doll was not used at this time.)

FIGURE 3

Find the picture

The child remains seated for the entire task and must imagine what Nellie would see as she is moved to each of the seven positions, not including "A," identified in **figure 5**. She would see picture number 3 (see **fig. 1**) from this position.



FIGURE 4

Find the picture

Nellie, the doll, would see picture number 5 (see **fig. 1**) from this position.



FIGURE 5

Find the place

The letters mark places around the table where the child is asked to imagine moving in order to view each picture in **figure 6**. The doll is not used in this task. The places labeled "A" are in clockwise order, at 45-degree intervals, beginning with position "A."



FIGURE 6

Find the place

The child is asked to select the place to which he or she would need to move to view each of these pictures. Note the round bases in these pictures, which are intended to prevent the use of sides and corners as visual cues. The pictures were shown in the order indicated below.

1st	2nd	3rd
4th	5th	6th
7th	8th	9th
	10th	



Katie

Assessment Questions and Tasks

Why did you pick number 9? Can you tell me how they look the same?

Let's look at the scene on the table and study it. The tree is to the left of the house, and the tree is in front of the tower. Can you use this information to help you find the right photo?

Student's Responses

[Selects number 9 (see **fig. 1**) to represent the scene she is looking at.]

Because they look the same. This is here and that is there. [Katie is referring to the fact that the tower, house, and tree are all in the picture. She is not attending to relative positions of the objects.]

[Number 3 (see **fig. 1**) is selected. Katie appears to be focusing on objects only. Relative positions are not important to her. When these are pointed out, she seems to have difficulty focusing on more than two objects at a time. In addition, her ability to use the vocabulary and thinking needed to work with relative positions is weak. We stop the assessment here.]

To gather some information about Katie's working style, we moved around the table, looking at the scene from different perspectives. She said she was surprised that the scene looked so different at different places. Katie dropped to her knees to keep the scene at eye level so she could see the objects (positions G, E, and D—see **fig. 5**). Pictures 6, 7, 3, and 4 were the most engaging for her (see **fig. 1**). Katie claimed that each was a match for position G. When informed that only one was the actual photograph, she resorted to physically moving the objects in the scene to match the pictures. Her proclaimed matches often were incorrect and I asked her to rethink her choices. The concepts of *closest*

to and *farthest from* were the most difficult for her. She never repeated the language I modeled for her; she only continued to say, "This goes here" or "It goes there."

Katie is struggling with the vocabulary and concepts of relative position. She needs to concretely move objects and she needs assistance in analyzing scenes. It is as though she does not consider these aspects to be important. A lot of real-life experiences in which the concepts of relative position are built and reinforced and the language consistently modeled will help Katie develop these skills. These experiences could include setting the table, playing games, or talking about objects in the room.

Mary

Assessment Questions and Tasks	Student's Responses
	[Selects number 10 (see fig. 1) for the scene she is looking at.]
Why did you pick number 10?	Because the tree is here, the house is here, and the tower is here.
Suppose I couldn't see what you are pointing to. Can you describe where the objects are so I can picture them in my mind? For example, the tree is to the left of the house and the tower is behind the tree.	OK. [Mary repeats my description.]
What if your doll [Nellie] were sitting here [the doll is placed at position C—see fig. 5] and she could see? Which picture do you think would show what she would see?	Number 2 [see fig. 1].
Why did you select that picture?	Because the tree has to be there and the tower there.
Can you describe it for me so I can see it in my mind?	I always have trouble with left and right.
Which hand do you write with?	[She holds up her left hand.]
Then just say the tower is on my writing hand side of the tree and the house is on the other side.	OK.
If your doll moved here [position D—see fig. 4] what do you think she would see?	Number 5 and number 11 [see fig. 1], but I can see part of the house.
Get on your knees and look at the scene. That is where the eye level of your doll and the camera are. Which of these pictures do you see now?	Number 5 because the tree is not on my writing hand side. [Still on her knees, she moves to position E (see fig. 3).] Why can't I see the tree? [She looks at the photos and picks number 3 (see fig. 1) for this new position. Then she stands up.] There it is. [Mary then moves to position G and becomes overwhelmed when she tries to choose between numbers 6, 7, 3, and 4 (see fig. 1). We stop the assessment here.]

Mary enjoys visual things. She often called attention to the patterns on the tower and the shape of the tree. She frequently picked up the objects and held them gently in her hands. Before the assessment began, she ran to me while I was setting up the scene to announce, "The pictures were photos of that." She uses some of the relative position vocabulary correctly, but is not yet proficient. She understands what the task is about and can often pick the correct photograph, but the analysis is taxing for her. She has to think very hard. Two objects are manageable, but having to worry about three objects is a stretch for her,

probably because she is still refining the concepts and vocabulary for relative position. However, even when she gets tired she remains interested. When the task becomes too taxing, she diverts attention to questions of interest to her, such as "Why can't I see the tree?"

Mary would profit from working with additional tasks like this one, using two objects and expanding it to three as her vocabulary and understanding become more proficient. She needs someone to work with her, modeling vocabulary and scene analysis, until her thinking for this task becomes consistent.

Sally

Assessment Questions and Tasks

If I put your doll [Nellie] here [at position E—see **fig. 3**] and she could see, which photo do you think would show what she would see?

Is this one hard to think about?

Would you like to work up to it with easier ones?

[The doll is moved to position C.]

Why did you select that picture?

[H, D, B, and F are presented.]

[The doll is moved to position E (see **fig. 3**).]

You have been correct on all your choices. Do you want to actually look at the scene and check this one?

Now we will change the task a little. I am going to give you a picture and I want you to tell me at which place you would see the picture. The letters around the table (see **fig. 5**) identify the places you can choose. [Pictures are delivered in the order indicated in **fig. 6**.]

Student's Responses

[Confidently selects number 10 (see **fig. 1**) to represent the scene she is looking at and correctly describes why she selected that photograph.]

[Sally cringes.]

[She nods.]

Yes.

[Sally selects number 2 (see **fig. 1**).]

Because the tree will be in front of the house and the tower will be to the left of the tree.

[All pictures are correctly chosen and confidently and accurately described.]

[Number 3 (see **fig. 1**) is selected.]

Yes. [She goes to the position and drops down on her knees to get at eye level. She smiles.] Yup.

[Sally chooses places F, B, H, and D for the first four pictures shown (see **figs. 5** and **6**).]

When Sally looks at the fifth picture, she says, “Can there be two pictures of the same place?” I ask her if she thinks there is a problem. Her response is, “Can I change an answer?” The last picture she had looked at is returned. She stares at the two pictures, perplexed. When asked if there is something wrong, she nods. “I may have put in a trick picture,” I say. “What do you think?” Sally replies, “Yes.” I ask her which picture is the right one and she replies, “The fourth picture I saw because the tree has to be on the right.” We call the fifth picture a photographically flipped image of the actual photograph. The remaining places chosen are A (for the sixth picture shown—see **fig. 6**), C (seventh), F (eighth), G (ninth), E flipped (tenth).

As soon as Sally had made choices for all the pictures, she was presented with each picture individually and asked to go to the places she had selected to see if she was right.

For the first picture, her choice had been “F.” She announced, “Not right—this is flipped. Did I pick another one for F?” I gave her the other selec-

tion she had made for F. “Yep,” she said. She pronounced the rest correct until the last one, for which she had chosen E flipped. For this one she said, “This is wrong, too. It is G flipped.”

Sally was confident, verbally proficient, and highly interested throughout the entire task. She demonstrated well-developed, stable schemas for working with perspective. Sally is ready to encounter tasks that use perspective concepts, such as interpreting drawings of cube structures.

I asked Sally why she thought I had brought this task for her to work on. She said, “To give us a mental puzzle. It requires you to think really hard about everything there.” When asked if she liked it, she said, “Yes, it was fun to have to think hard.”

Making the Activities

It helps to select objects of varying heights so that one of the objects is not visible from at least one perspective. When taking the pictures for the cards, be sure to use a tripod so that all the pictures are

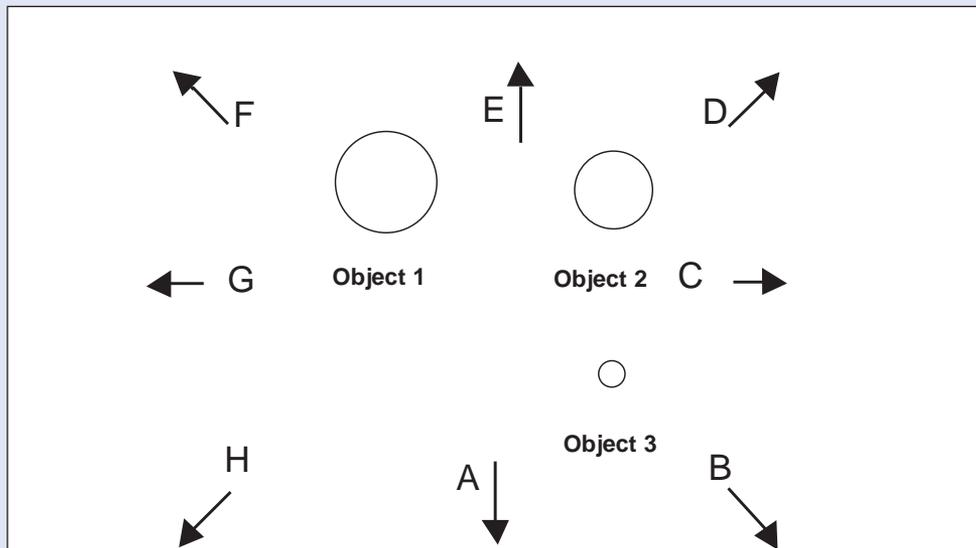
Sample task card for practice sessions

The reading level of the task card can be adjusted to fit the age of the students.

FIND THE PLACE (see figs. 5 and 6)

Materials	Objects, direction board, picture packet (numbered pictures), recording sheet, place markers (different letters)
Preparation	<ol style="list-style-type: none"> 1. Find a partner. 2. Collect the objects and set up the scene according to the direction board. 3. Put your names on your recording sheet. 4. Sit at the starting position with the picture packet (see fig. 6).
Directions	Stay seated at the starting position. Select a picture and imagine where you would need to move in order to see that picture. Record your choice on your recording sheet. Have your partner do the same and discuss your selections. When you have finished your discussion, move around to the place you selected. Does the view match the picture you selected? If not, why not? Record your findings. Try the next picture.
Variations	<ol style="list-style-type: none"> 1. Make all your selections first. Then check each one. 2. Have your partner set up the scene according to the direction board when you are not looking. Then, while sitting at the starting position, choose the places for all the pictures in the packet. Record your selections. When you have finished, go around the board with your partner and check each of the scenes to see how close you were. 3. Choose a picture and tell where you would need to sit to see that picture. You and your partner can make choices together and then check.

Sample direction board



taken from the same height. Using height as a variable would make a good extension for children in the upper grades.

Digital cameras work well for this activity. Once your pictures are printed, you can develop activities that focus on each of the abilities previously described. For variety, you can make different sets of the activities using different objects.

After all the students' levels of understanding have been determined, and practice activities that are developmentally appropriate have been created, you can model for the whole class how to work with these practice activities and then place the task cards at a center, or some place where groups of children can work on them at their own pace.

Children should be encouraged to discuss their

FIGURE 9

Sample recording sheet for practice sessions

FIND THE PLACE (see **figs. 5 and 6**)

Name _____

Partner _____

Identification for each picture we used	Place I selected	Place my partner selected	Were we correct?

ideas while they are working on these tasks. Vital to their learning is that their ideas are challenged and they are required to defend them or even change their minds or revise their thinking.

It takes time for vocabulary and understanding

to develop. Working on the activities several times for short periods that are widely spaced throughout the year can be beneficial. The teacher needs to assess each student’s progress at regular intervals and adjust the activities accordingly. **Figures 7, 8, and 9** present a sample task card, a direction board, and a recording sheet.

These case studies typify the levels of understanding that I have observed among both young children and older students throughout my long career in mathematics education. My observations have made me cautious in assuming that students acquire an understanding of perspective from casual real-life experiences.

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