Laying the Foundation for Computational Fluency in Early Childhood

Sharon Griffin

Educators tend to think of computational fluency and number sense as two different types of mathematical knowledge. Computational fluency seems to entail a well-practiced and efficient use of procedures to compute how many items are in a set or how many there will be if sets are joined or separated. Number sense seems related to a deep understanding of the meaning of numbers. Through the phrasing of its major curriculum recommendations, NCTM may have unintentionally reinforced the tendency to think of these two types of knowledge as distinct or as opposed in some fundamental way. An oppositional frame of mind about these forms of knowledge is prompted by the suggestion that mathematics teaching should move away from an emphasis on facts and procedures and toward a focus on number sense (NCTM 1989), as well as a more recent recommendation that number sense should remain the dominant focus in mathematics teaching but computational fluency should not be neglected (NCTM 2000). One begins to wonder whether the two types of knowledge are acquired in different contexts and whether they require different methods of teaching. Nothing could be further from the truth.

Sharon Griffin is associate professor of education and psychology at Clark University in Worchester, Massachusetts.

Edited by **Julie Sarama,** *jsarama@buffalo.edu, and* **Douglas Clements,** *clements@buffalo.edu, State University of New York at Buffalo, Buffalo, NY 14260. This department addresses the early childhood teacher's need to support young children's emerging mathematics understandings and skills in a context that conforms with current knowledge about the way that children in prekindergarten and kindergarten learn mathematics. Readers are encouraged to send manuscripts for this section to "Early Childhood Corner," NCTM, 1906 Association Dr., Reston, VA 20191-1502.*

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Computational Fluency and Number Sense Go Hand in Hand

Recent research on the development of mathematical knowledge is strong evidence that computational fluency and number sense are intimately related. They develop together, and one cannot exist without the other (Griffin, Case, and Siegler 1994; Griffin, Case, and Capodilupo 1995; Griffin and Case 1997). Consider the following example. By the end of kindergarten—at about the age of 6—many children are at least able to tackle the following computation problem, which is presented orally, without manipulatives: "If I give you four chocolates, then I give you three more, how many will you have altogether?" What strategies do preschool children use to solve this sort of problem? What knowledge (number sense) underlies their strategy choice? In research conducted with this problem, which appears as an item on the Number Knowledge Test (see Griffin and Case 1997; Griffin, in press), five levels of strategy choice were identified for children between the ages of 3 and 6. The levels are presented below in the order in which children typically acquire them.

Level 1. The youngest children in the sample—most of them 3- and 4-year-olds—made no attempt to solve the problem but rather responded with blank expressions and verbalizations such as "I don't know" or "I haven't learned that yet." This suggested that they had not yet acquired the computation skills and number sense they needed to tackle this sort of problem.

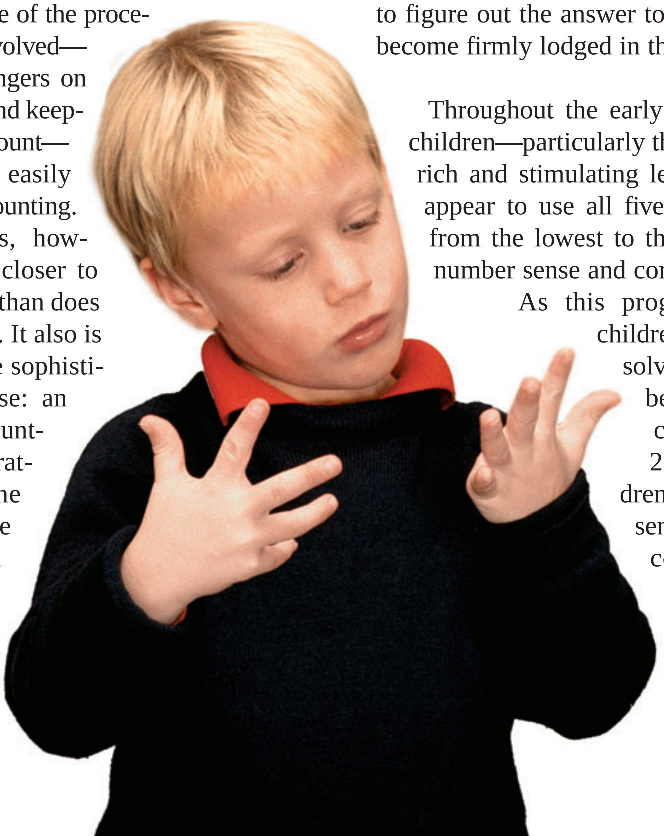
Level 2. At this level, demonstrated by some 4- and 5-year-olds, children do not use computation, such as counting, to figure out the answer. However, they do offer a solution that is usually tentatively phrased, indicating that it is a guess; for example, "lots," "five," or "ten." Although these responses are incorrect, they are reasonable answers. Children rarely choose a number less than five, which suggests that they have some understanding of addition; for example, they know that the answer has to be greater than four. They also have some number sense; for example, they know, at least intuitively, that "lots," "five," and "ten" refer to quantities that are greater than four.

Level 3. At this level, demonstrated by many 5-year-olds, children use the "count up from one" strategy, with or without using their fingers, to figure out the answer. They often hold up four fingers on one hand, touching each finger and counting as they do so, then hold up three fingers on the other hand, counting from one again. Finally, they count all the fingers that they have raised, touching their noses or nodding their heads to keep track of the count because the fingers of both their hands are occupied with the task of marking the items that they count. Because of the procedural challenges involved—such as keeping fingers on both hands raised and keeping track of the count—this strategy can easily produce errors in counting. The strategy does, however, get children closer to the correct answer than does the level 2 strategy. It also is evidence of a more sophisticated number sense: an awareness that counting is a useful strategy to determine how many items are in a set or in a combined set that can be represented in some way, such as with fingers or with mental images of chocolates.

Level 4. At this level, demonstrated by many 5- and 6-year-olds, children use the more sophisticated "count on" strategy. They start their count at four; then they say the next three numbers in the sequence, often using their fingers to keep track of how many numbers they have said, and give the last number they have said as the answer. This strategy is faster and simpler, procedurally, than the level 3 strategy and usually yields the correct answer. The strategy depends, however, on a fairly sophisticated number sense that has been referred to as a "central conceptual structure for number" (Griffin and Case 1997). For example, children must know that they do not need to count objects to figure out how many there are because (a) the number 4 always refers to a set of four things; (b) the next counting number in the sequence means that a set has been increased by one; and (c) all they need to do to solve the problem is to start counting at four and count up three more numbers. They do not even need to imagine real objects. This strategy and the knowledge that underlies it allow children to solve any single-digit addition problem that is presented to them, provided that they can count accurately.

Level 5. At the highest level, children use a "retrieval" strategy. They respond quickly, pulling from memory the correct answer to the question, with no need for computation. When asked how they figured out the answer, they say, "I just knew it" or "It was in my head." Children using this strategy have not simply memorized the addition facts; they have used the level 4 strategy so many times to figure out the answer to 4 + 3 that the sum has become firmly lodged in their memory.

Throughout the early childhood years, most children—particularly those who have access to rich and stimulating learning environments—appear to use all five strategies, progressing from the lowest to the highest level as their number sense and computation skills mature. As this progression demonstrates, children use number sense to solve the sample problem before they attempt to use computation (see level 2). Increases in children's developing number sense prompt them to use counting to solve the problem (see level 3). Substantial practice in their use of counting to solve problems of



this sort leads to increases in their number sense and the development of more sophisticated computation strategies (see level 4). The effortless retrieval of number facts demonstrated at level 5 is therefore the result of a lengthy developmental process that has occurred over a number of years.

Teaching Computational Fluency and Number Sense

How should teachers teach these strategies to children who may not acquire them on their own? The strategies are not directly taught in the Number Worlds program, an early mathematics program that I developed with my colleague Robbie Case (see Griffin and Case 1996 for the kindergarten level of this program and Griffin 2000 for the preschool level). Instead, teachers carefully nourish the knowledge that underlies the strategies by exposing children to a series of games and activities that parallel their natural developmental progression. This enables children to start at a level they are comfortable with and to progress through the sequence at their own pace. Three important objectives of the program and activities to teach them are described below.

Knowing the number sequence from 1 to 10. If children are going to use counting to solve problems (see the level 3 strategy), they must memorize the number sequence and be able to use it with ease and accuracy under a variety of conditions. Repeated practice with the following activities for the number sequence from 1 to 5, and later for 1 to 7 and then 1 to 10, can encourage counting fluency:

- Have children count after you.
- Have children take turns counting (saying the number sequence) by themselves.
- Have children say the sequence serially, with one child saying the number 1 and the next child saying the number that comes next, and so on.
- Have one child start counting and stop when the teacher winks or claps. The next child will continue counting where the first child stopped.
- Have children play “Catch the Teacher” and identify counting mistakes that you deliberately make, such as skipping a number or saying a number twice.

Understanding that each number in the sequence refers to a set of a particular size. In order to be sure that the results of counting will always tell them how many, no matter what size the objects are or how the units are represented, children need a lot of practice counting sets that are represented in a variety of ways; for example, as objects, dot-set patterns, positions on a path, or points on a dial. The following strategies will encourage fluency in identifying set size:

- Have children count or create sets of different sizes in a variety of contexts; for example, count four versus six objects or walk four versus six steps along a numbered path.
- Encourage children to talk about the results of these procedures in as many ways as they can. For example, they might say, “I’m standing on the number 4 on the path, and I still have a long way to go to the end.” These discussions will help children make sense of the procedures they have just enacted.
- Encourage children to talk about how sets of different sizes produce differences in magnitude, using the correct terminology for each context. For example, a set of six cookies is *bigger than* a set of four cookies; six steps is *farther along* the path than four steps. These discussions will help children make mathematical or quantitative sense of the procedures they have just enacted.

Children will learn from these activities that a set of six, for example, is always larger than a set of four, no matter how it is displayed, and this difference is always precisely the same amount: two units. Understandings such as these are the foundations of number sense and will enable children to not only count or compute more fluently but also understand why it is useful to do so in a variety of contexts to solve a wide range of problems.

Knowing that the next-highest number in a sequence means that a set has increased by one. In order to use the counting numbers alone to solve problems, without the need for any object representations (see the level 4 strategy), children must know how moves up and down the counting sequence affect increases and decreases in quantity. A popular activity from the Number Worlds program that fosters this understanding is the “Plus Pup” game. To play this game, children watch and count as the teacher places a certain number of cookies in a lunch bag. As the teacher walks across the room, Plus Pup, a puppet wearing a card displaying “+ 1,” puts one more cookie in the bag. Children are asked to solve the following problem: “How many cookies are in the bag now?” After children make their predictions and justify them, the teacher opens the bag and reveals the contents. The class then discusses correct and incorrect predictions and the procedures that children used to generate each. Repeated exposure to this activity with different quantities of cookies and a more generous Plus Pup bearing “+ 2” gives children a sense that counting up from the initial quantity is an effective strategy to use to determine the results of adding one or two items to a set. When children understand the meaning of a computation strategy, they will use it more frequently and acquire greater fluency.

In all the activities described above, children are encouraged to count out loud, use their fingers for as long as they wish, describe the process and the results of their computation activity in everyday language and in the more formal language of mathematics, and explain their reasoning by answering questions such as “How do you know? How did you figure that out?” Although the computation and number sense activities in this article are simple, the knowledge base that they foster—the central conceptual structure for number—is complex. In this knowledge base, computation procedures such as counting are firmly embedded in a network of meaning that intimately links computing and sense making. This knowledge base enables children to solve beginning addition problems with ease and efficiency, and it serves as an essential foundation for the development of the more advanced computation skills and number sense that children will need in the coming years.

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