Thank you for your purchase –

**Please be sure to save a copy of this activity to your computer!**

# JUMPING TO SOLUTIONS

**Topic**
Solving equations

**Key Questions**
1. How can you determine the length of a jump by knowing the position of the jumper at two different times?
2. How does an equation describe how a person moved to a position?

**Learning Goals**
Students will:
- measure the position and number of jumps of a person,
- generate an equation that describes how the person got to the end position, and
- determine the length of the person's jump from the equation.

**Guiding Documents**
*Project 2061 Benchmarks*
- *An equation containing a variable may be true for just one value of the variable.*
- *Mathematical statements can be used to describe how one quantity changes when another changes. Rates of change can be computed from magnitudes and vice versa.*
- *The operations + and – are inverses of each other—one undoes what the other does; likewise x and ÷.*

*NCTM Standards 2000**
- *Develop an initial conceptual understanding of different uses of variables*
- *Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships*
- *Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations*

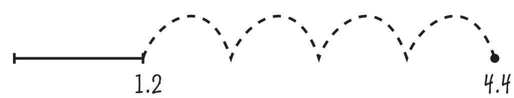
**Math**
Algebra
  solving equations

**Integrated Processes**
Observing
Comparing and contrasting
Generalizing

**Materials**
Metric tape measure
2 markers (blocks, erasers)
Student pages

**Background Information**
Students need a concrete experience with which to relate their thinking. Jumping, hopping, and leaping provide very active experiences to which students relate their thinking. Having a student start jumping at a random position along a tape measure and then measuring the student's position after a number of jumps provides the context for an equation. This meaningful context allows students to develop a conceptual understanding of the meaning of an equation and how to solve it.

Consider the example where a student starts at the 1.2-meter mark on a tape measure, takes four jumps, and is at 4.4 meters. When students are asked to draw a sketch and describe how the jumper got to the 4.4-meter position, their sketches look like this.




They explain, "The jumper started at 1.2 meters and jumped four times to end at 4.4." This description clearly translates into the equation $1.2 + 4J = 4.4$ where $J$ represents the length of the jumps. It is not hard for students to recognize that this is equivalent to the form $4J + 1.2 = 4.4$ found in their math texts. This form might be translated "four jumps and 1.2 meters is 4.4 meters."

Now the question "What is a jump ($J$)?" or "How long is a jump?" naturally arises. When students are posed this question, they need to be given time to ponder how this can be determined. Because of the experience and context surrounding the question, students can generally grapple with the numbers to come up with the solution. As students share their thinking, they will clarify what they did and will help those students having difficulty. When students clearly understand what they are doing, it can then be modeled in algebraic form. As students work through the context numerous times, they will be able to deal with more abstraction until they can work similar problems symbolically with no reference to a context.

Following is a typical discussion in a classroom:

**Teacher:**
"Now we have the equation about Patrick's jumping—$1.2 + 4J = 4.4$ The $J$ in the equation represents jumps. So how long is a jump? Use your sketch, numbers, or an equation to figure this out." The teacher than allows students ample time to grapple with the question on their own. As students start to find the solution, the teacher suggests they share with a partner their methods of solution. This allows others time to find a solution or provides a chance for the struggling student to make progress toward a solution. When it is evident that most of the class has found a solution, the teacher asks, "How many of you found a solution or think you have made a good start?" As most or all of the class raises their hands, the teacher has encouraged participation. "Great! Who knows what they did to get started?" Almost all the hands remain up, and the students are feeling successful. "Who would like to share with the class what they did to get started?" By asking for volunteers, no one is put on the spot and all the students remain engaged.
**Danny:**
Crosses out the beginning of the tape on the sketch on the board and says, "I got rid of this."
**Teacher:**
"Why did you do that?" is asked to see what Danny is really thinking.
**Danny:**
"We want to know about the jumps, so we need to get rid of the 'head start.'"
**Teacher:**
"How many understand why Danny wants to get rid of this starting position?" Most students respond with a hand. "How many of you did something like Danny and got rid of the starting distance?" Some students raise their hands while others look puzzled. "Did you all do what Danny did and just cross out the start, or is there something to do with the numbers?
**Janet:**
"I just took away the starting number and got 3.2."
**Teacher:**
"Can you write that as a number sentence?" Janet writes $4.4 - 1.2 = 3.2$. "Thanks, Janet. Now does everyone see where Janet got this number? Why is she doing this?"
**Miguel:**
"She has the finish and she is taking away the 'head start.'"
**Teacher:**
"So, what does Janet's answer of 3.2 tell us about Patrick's jumping?"
**Katie:**
"It's how far he jumped."
**Teacher:**
"So, Danny just crossed out the head start and Janet did an arithmetic problem. Can anyone explain how what Danny and Janet did are related?"
**Olivia:**
"Danny crossed out the head start to get the jump part and Janet subtracted the head start so it's only the jumps."
**Teacher:**
"Now we have seen how we can use the pictures or numbers to think about the problem, but we are supposed to be learning some algebra, so let me show you how we can think about what we have done with algebra. Here's the equation. Now Danny says get rid of the head start." The teacher crosses out the 1.2 in the equation.

$$\cancel{1.2} + 4J = 4.4$$

"But you can't just cross out numbers you don't want in the problem. To get rid of it, you need to subtract it. That's what Danny was thinking about, so we just do the subtraction on the description side of the equation."

$$\begin{array}{rcl} \cancel{1.2} + 4J & = & 4.4 \\ \underline{-\,1.2} \quad\quad & & \\ 4J & = & \end{array}$$

"Now, if you are going to get rid of the head start on the description side, you have to make the distance jumped shorter too. That's what Janet was doing with her subtraction."

$$\begin{array}{rcl} \cancel{1.2} + 4J & = & 4.4 \\ \underline{-\,1.2} \quad\quad & & \underline{-\,1.2} \\ 4J & = & 3.2 \end{array}$$

"So we took off the head start and know that four jumps are 3.2 meters. Now, how did you determine how long each of the four jumps are?"
**Laura:**
"The jumps in the picture split the distance into four pieces, so you split the 3.2 into four pieces too."
**Teacher:**
"So how do you split up numbers into four pieces?"
**David:**
"You divide 3.2 by four to get a jump. I got 0.8."
**Teacher:**
"Does everyone see why the four jumps split the distance into four equal pieces and why David divided 3.2 meters four ways to get a jump length? In algebra we can show this too. Laura suggested the four jumps divided the distance, so we divide the four jumps four ways. David divided the distance of 3.2 four ways too to get the distance of each jump."

$$\begin{array}{rcl} \cancel{1.2} + 4J & = & 4.4 \\ \underline{-\,1.2} \quad\quad & & \underline{-\,1.2} \\ 4J & = & 3.2 \\ \underline{\div 4} & & \underline{\div 4} \\ J & = & 0.8 \end{array}$$

To confirm the correct solution, the teacher brings the students' attention back to the tape measure. From the starting position, four 0.8 meters "jumps" were made to see that it did get the jumper to the 4.4-meter position.

As students work through problems thoughtfully at a conceptual level, they recognize why they are doing things mathematically and see the connection to the abstract algebra manipulation. Working through a number of similar problems in this context, students begin to translate equations with respect to the context and can reason to the solution with the support of the context. With continued practice, students move to abstract manipulation with no reference to the context but can always rebuild their understanding through the context.

Subtraction is introduced into equations by having the hopper start from a position before the start of the tape measure. A hopper starting at 0.6 meters before the tape and taking five hops to reach 2.9 meters is represented by the equation: $-0.6 + 5H = 5H - 0.6 = 2.9$. With this situation, the hopper hopped the 2.9 on the tape plus the 0.6 meter before the tape. Add 0.6 to both sides to show that $5H = 3.5$. Again a division by five expresses the length of a hop ($H = 0.7$).

**Management**

1. The animation *Part Two: Multiple Representations* is on the accompanying DVD. It is also available at the following URL: www.aimsedu.org/media/books/. Make preparations in your classroom so students can view the animation. If viewing through a computer, a projector enhances the experience.
2. An area for students to jump needs to be cleared before the activity. A wide aisle in the front or middle of the classroom works well. The area needs to be long enough for students to take seven to 10 jumps, hops, or leaps.
3. The best success is achieved when students are allowed to grapple with the situation and develop a meaningful solution on their own. Allow time for students to consider the problem and share their reasoning. If students make no progress on their own, have them follow the leading questions on the student page.
4. The best results come when the lengths of the jumps, hops, or leaps are consistent. Encourage the students not to go for the longest jump but to make the most consistent jumps. Different forms of jumping are listed on the student page to suggest that the value of the jumps should change and different variables would be used for each. The definition of each type can follow the illustration. A jump is on one foot; a hop uses both feet; and leaping involves bounding on alternating feet.

**Procedure**

1. Stretch out a tape measure across the length of the prepared jumping area.
2. Select three volunteers from the class—a jumper and two markers.
3. Place the jumper at a random position on the tape measure and have one of the volunteers place a marker behind the jumper's heel.
4. Have the jumper take three to seven jumps down the tape measure and have the second volunteer place a marker at the jumper's heels.
5. Instruct students to complete the sketch of the jump by starting on the X and drawing in the number of jumps taken. Have them record the measurements of the starting and ending positions.
6. Ask students to verbally describe how the jumper got to the ending position and develop an equation that is equivalent to the description.
7. Have students contemplate how to determine the length of each jump. If students make no progress on their own, pose leading questions using the student page.
8. Have students share with the class the different methods they used to determine the size of a jump.
9. Using a similar procedure, have students reinforce their thinking by measuring a hopping and leaping student. If it is appropriate, have the third hopper start before the tape measure so the students will be introduced to subtraction in the equation.
10. With the investigation as a context, have students make sketches and equations to solve similar contextual problems on the student pages.

**Connecting Learning**

1. Describe how the person got to the finish. [Beginning at the start position, the jumper made a number of jumps to get to the finish.]
2. How would you translate your verbal description into an equation? [start position + number of jumps = finish position]
3. Using your sketch, equation, or numbers, how would you determine the length of a jump?
   a. How many jumps did the jumper make?
   b. Where did the jumper start?
   c. Where did the jumper finish?
   d. How far did the jumper jump all together?
   e. How far did the jumper jump each time he or she jumped?
4. What steps did you follow every time to determine the length of a jump, hop, or leap?

**Extensions**

1. Have students write their own narratives about jumping situations and make a sketch, equation, and solution. Then have students share their narratives with each other, solve, and check.

2. Have students look at the two-step solving equations in their texts. Have them discuss the similarity between the equations in the book and equations from the situations. Have them translate the equations into jump situations and solve.

**Solutions**

*Page Three*

1. $12 + 5J = 27$
   $5J = 15$
   $J = 3$
2. $18 + 7H = 53$
   $7H = 35$
   $H = 5$
3. $12 + 9L = 39$
   $9L = 27$
   $L = 3$
4. $-14 + 12J = 10$
   $12J = 24$
   $J = 2$
5. $5 + 6H = 35$
   $6H = 30$
   $H = 5$
6. $7L + 6 = 20$
   $7L = 14$
   $L = 2$

*Page Four*

1. $15 + 4J = 43$
   $4J = 28$
   $J = 7$
2. $5H + 9 = 29$
   $5H = 20$
   $H = 4$
3. $-8 + 9L = 19$
   $9L = 27$
   $L = 3$
4. $23 + 3J = 35$
   $3J = 12$
   $J = 4$
5. $-10 + 6L = 20$
   $6L = 30$
   $L = 5$
6. $6H + 13 = 55$
   $6H = 42$
   $H = 7$
7. $45 + 7H = 59$
   $7H = 14$
   $H = 2$
8. $24 + 9L = 42$
   $9L = 18$
   $L = 2$
9. $6J + 15 = 33$
   $6J = 18$
   $J = 3$
10. $-5 + 4H = 23$
    $4H = 28$
    $H = 7$
11. $7J - 6 = 15$
    $7J = 21$
    $J = 3$
12. $3L - 7 = 20$
    $3L = 27$
    $L = 9$

* Reprinted with permission from *Principles and Standards for School Mathematics*, 2000 by the National Council of Teachers of Mathematics. All rights reserved.

SOLVING EQUATIONS: A CONCEPTUAL APPROACH

# JUMPING TO SOLUTIONS

## Key Questions

1. How can you determine the length of a jump by knowing the position of the jumper at two different times?
2. How does an equation describe how a person moved to a position?

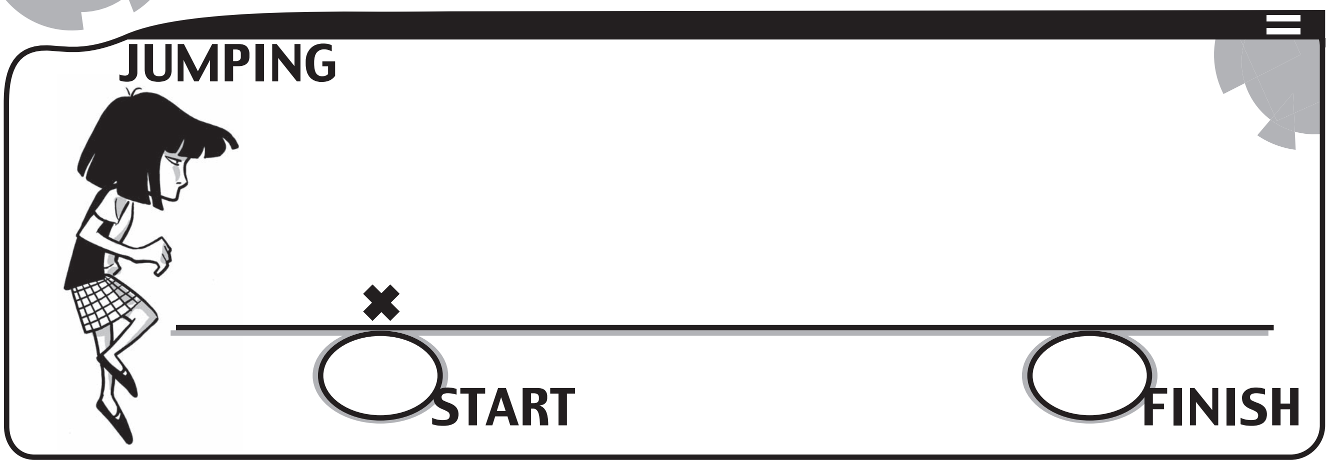
## Learning Goals

**Students will:**

- measure the position and number of jumps of a person,
- generate an equation that describes how the person got to the end position, and
- determine the length of the person's jump from the equation.

# JUMPING TO SOLUTIONS




1. Make a sketch of the jumps on the tape measure and record the start and finish position of the jumper.

2. Complete the sentence:

   The jumper starts at ______ centimeters, takes ______ jumps,

   and finishes at ______ centimeters.

3. Translate the sentence into an equation. Use *J* to represent a jump.

4. Use the sketch, numbers, and equation to determine the length of each jump.
   a. How far did the jumper move from start to finish? How do you get this number?

   b. How long was each of the jumps? How do you get this number?

# JUMPING TO SOLUTIONS

## HOPPING


1. Make a sketch of the hops on the line and record the start and finish position of the hopper.
2. Complete the sentence:

   The hopper starts at ______ centimeters, takes ______ hops, and finishes at ______ centimeters.
3. Translate the sentence into an equation. Use *H* to represent a hop.

4. Use the sketch, numbers, and equation to determine the length of each hop.

## LEAPING


1. Make a sketch of the leaps on the line and record the start and finish position of the leaper.
2. Complete the sentence:

   The leaper starts at ______ centimeters, takes ______ leaps, and finishes at ______ centimeters.
3. Translate the sentence into an equation. Use *L* to represent a leap.

4. Use the sketch, numbers, and equation to determine the length of each leap.

# JUMPING TO SOLUTIONS

For each situation, make a sketch, write an equation, and determine the length of the jump, hop, or leap.

1. The student stands at the 12-foot mark and makes five jumps to finish at the 27-foot mark.

2. The frog is at the 18-inch mark and takes seven hops to finish at the 53-inch mark

3. The kangaroo is at the 12-yard line and takes nine leaps to finish at the 39-yard line.

4. The flea is 14 millimeters behind the start of the ruler and makes 12 jumps to finish at the 10-millimeter mark.

5. Starting at the five-foot mark, the grasshopper makes six hops to finish at the 35-foot mark.

6. The leaping lizard makes seven leaps to move from six inches to 20 inches.

# JUMPING TO SOLUTIONS

Each of the equations describes a jumping, hopping, or leaping situation. Translate the equation back into English or make a sketch if it helps you make sense of the situation. Then use the equation to determine the length of a jump, hop, or leap.

1. $15 + 4J = 43$
2. $5H + 9 = 29$
3. $-8 + 9L = 19$
4. $23 + 3J = 35$
5. $-10 + 6L = 20$
6. $6H + 13 = 55$
7. $45 + 7H = 59$
8. $24 + 9L = 42$
9. $6J + 15 = 33$
10. $-5 + 4H = 23$
11. $7J - 6 = 15$
12. $3L - 7 = 20$

# JUMPING TO SOLUTIONS

## Connecting Learning

1. Describe how the person got to the finish.

2. How would you translate your verbal description into an equation?

3. Using your sketch, equation, or numbers, how would you determine the length of a jump?
   a. How many jumps did the jumper make?
   b. Where did the jumper start?
   c. Where did the jumper finish?
   d. How far did the jumper jump all together?
   e. How far did the jumper jump each time he or she jumped?

4. What steps did you follow every time to determine the length of a jump, hop, or leap?