Are We Overemphasizing Manipulatives in the Primary Grades to the Detriment of Girls?

In keeping with the Equity Principle of the *Principles and Standards for School Mathematics* (NCTM 2000), educators must maintain high expectations for all children and continually examine their practices to ensure that all children learn mathematics with understanding. The instructional practice of using manipulatives for problem solving merits closer examination because it may send the wrong message to some children. Recent research indicates that some girls' understanding seems to be limited by their overreliance on manipulatives. Before presenting the research findings, I will outline the role of manipulatives in supporting the development of children's understanding, then examine how this promising practice can be detrimental when used too often.

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Background Information

During the last twenty-five years, arithmetic curricula have included more hands-on activities with an increasing emphasis on problem solving. According to the Representation Standard in *Principles and Standards*, "instructional programs from prekindergarten through grade 12 should enable all students to use representations to model and interpret physical, social, and mathematical phenomena" (NCTM 2000, p. 136). Representations, such as manipulatives, help children develop understanding by building a foundation for the later use of symbols. When given a choice of how to solve a problem, children gravitate toward manipulatives because they can act out the situation or relationships in the problem.

When children are free to choose strategies for problem solving in single-digit arithmetic, they typically progress from using concrete strategies to more abstract strategies. For example, consider the following problem: "Lisa had 8 seashells in her collection. Hal gave her 5 more seashells. How many seashells did Lisa have then?" To solve this problem, children in the beginning stage of development will make a set of 8 objects and another set of 5 objects, join the two sets, and count all the objects. As they progress in their development, children do not need to build the first set but can count on from 8 to get the answer. Eventually, children can solve the problem abstractly without requiring a concrete model of either quantity in the problem (see Carpenter et al. [1999] for a complete discussion of these developmental stages). This progression of strategies is quite natural, and even five-year-olds move from counting-all strategies to counting-on strategies without instruction (Groen and Resnick 1977).

Some children progress along the same kind of developmental path for multidigit addition and subtraction, initially using concrete strategies in which they manipulate the quantities in the prob-

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lem, then using more abstract strategies. If they are permitted to solve problems using any technique they like, these children will initially use manipulatives. While continuing to solve problems, they develop knowledge of the structure of the base-ten number system and become acquainted with the principles underlying addition and subtraction. They can then use basic principles to work more abstractly, either mentally or with number symbols on paper. When this developmental sequence unfolds, the children have a deep understanding of base-ten numbers and can transfer their understanding to difficult problems (Carpenter et al. 1998). Unfortunately, not all children progress toward the use of abstract problem-solving strategies. This article examines why some children continue to use more cautious approaches.

Mental Mathematics versus Block Mathematics

To understand children's differing developmental paths, consider two second graders who were participants in a larger study of children's invented strategies (Ambrose 1998). These children were given the following problem: "There were 91 second graders and 37 first graders on the softball team. How many more second graders than first graders were on the softball team?"

Paul solved the problem mentally, saying, "Ninety take away 30 is 60; take away 7 is 53. Add that 1 from the 91, and you have 54." Paul had developed this unusual approach on his own while solving numerous word problems involving multidigit numbers. Before developing this strategy, Paul had solved many problems using base-ten blocks and hundred charts. His strategy was complicated because it required him to keep track of which numbers he had to subtract and which numbers he had to add. Because Paul had devised the strategy himself, he could keep track of the numbers and generate an accurate answer. Some mathematics educators would say that Paul used an "invented strategy," whereas others would say that he used "mental mathematics." All would agree that he used an unconventional but legitimate technique to solve the problem (see Fuson et al. [1997] for a detailed analysis of the kinds of strategies that children invent).

May, a girl in Paul's class, solved the problem by making a long train with 91 base-ten blocks. She then put a train of 37 blocks next to her train of 91. She painstakingly counted the difference between the two trains by ones and obtained the correct answer. Her teacher asked her if she could use tens, and May counted the difference between the trains by tens, getting the correct answer again.

May and Paul were at similar points in their mathematical development, performing similarly on a test of multidigit addition and base-ten knowledge, which put them in the middle of their second-grade class. May's other academic performance was higher than Paul's. She was a competent reader and writer and had strong work habits. Paul was a hesitant reader and struggled to complete assignments. In their classroom, the teacher encouraged them to develop solution strategies that they understood. The children were allowed to use whatever tools they wanted and were always expected to explain their thinking. Both Paul and May had each solved the same number of word problems. May typically used concrete approaches in which she often counted by ones. Paul typically used abstract approaches in which he broke numbers into tens and ones.

Research findings: Concrete strategies versus abstract strategies

I bring up May and Paul to illustrate a larger phenomenon found in several research studies. May's concrete strategy tends to be the approach that many girls choose. In a recent longitudinal study of first- through third-grade students, girls tended to use concrete strategies and boys tended to use abstract strategies on multidigit addition and subtraction problems (Fennema et al. 1998). The children in the longitudinal study were in problem-

centered classrooms in which students had opportunities to choose any strategy they could use with understanding. In a different study, first-grade girls tended to use manipulatives to enact counting-all or counting-on strategies, but boys tended to use retrieved or derived facts (Carr and Jessup 1997). This research indicates that girls often adopt concrete strategies and use them exclusively.

Research implications

To examine the implications of these research findings, again consider May's choice of blocks to solve the comparison problem. On the one hand, May's use of the concrete materials made perfect sense. She had the opportunity to build 91 and 37, and when she did, she discovered that 91 had 9 tens and a one. She was exposed to different interpretations of subtraction by counting the difference between the numbers. Her work with the manipulatives seems to have built a firm foundation on which future work will grow. Many teachers would be happy with May's sound approach to the problem.

On the other hand, May might have been operating on "automatic pilot." She may have been practicing a technique that she had perfected months ago, one that always worked to find an answer and that she could easily explain. In fact, May said that she had been using this matching strategy for this type of problem since kindergarten. When May did not have access to manipulatives, she drew tally marks and tended to count them by ones. Her modeling strategies were so ingrained that she used them most of the time. Such automatic execution of a concrete strategy, however, could present a danger. May never used her concrete approach to build more sophisticated strategies but became "stuck" in her use of the concrete. Her mathematical thinking did not progress as expected.

At the conclusion of the longitudinal study (Fennema et al. 1998), the researchers categorized the children according to the kinds of strategies they used over the three years. All the children used concrete strategies to solve addition and subtraction problems in the first grade. Fifty-three of the children, categorized as the *invented-strategy group*, went on to develop abstract invented strategies before adopting standard algorithms, typically during the second-grade year. Sixteen children, categorized as the *standard-algorithm group*, began using standard algorithms immediately after giving up concrete strategies, also during second grade. The standard-algorithm group rarely used invented strategies, even though their use was encouraged in the students' classrooms. Standard algorithms were introduced into the classroom by children who learned them at home. The teachers allowed their use but encouraged the children to use alternative strategies as well. The children who used standard algorithms were largely unsuccessful when asked to solve the following two extension problems (see **table 1**):

- Ellen had 4 dollars. She spent 1 dollar and 86 cents for a toy. How much money did Ellen have left?
- Gene had $398. How much more would Gene need to save to have $500?

The researchers hypothesized that the invented strategies helped the children who used them develop conceptual understanding that they could apply flexibly to difficult problems. The standard-algorithm group did not have the same flexibility or conceptual understanding.

Almost all the children in the algorithm group were girls (see **fig. 1**), but not all the girls were in this group. About half the girls used the same kind of invented strategies that their male peers used. Interestingly, only two of the sixteen children in the standard-algorithm group were boys. The interaction between gender and strategy use is clear, but it does not affect all girls.

Both concrete strategies and standard algorithms can be used without much reflection. The girls who used these automatic-pilot strategies had limited understanding, as evidenced by their diffi-

TABLE 1

Performance on two extension problems in spring of grade 3 (numbers of children)

Student Groups	Total in Group	Solved Both Problems Correctly	Solved One Problem Correctly	Solved Neither Problem Correctly	Mean Score (Possible Score of 2)
Invented-strategy users	53	27 (51%)	16 (30%)	10 (19%)	1.32
Standard-algorithm users	16	2 (12%)	3 (19%)	11 (69%)	0.43

culty with the extension problems. In the remaining sections of this article, I consider explanations for some girls' reliance on concrete strategies and standard algorithms and suggest some interventions that may prompt girls to use a wider range of strategies.

Potential Explanations

Perhaps girls in primary grades tend to use concrete strategies because they think teachers expect them to do so. Teachers may encourage the use of concrete-based strategies when they ask such questions as: "Can you show me how you did that problem?" or "Can you solve that problem with the blocks?" Some girls are particularly sensitive to the directions of the teacher and try to comply with expectations whenever they can. Such questions send a message to these girls that concrete strategies are what the teacher wants. In later grades, teachers sanction other conventional strategies that girls adopt and use to the exclusion of more intuitive approaches. Girls seem to learn what is taught, and concrete, conventional strategies are often emphasized in instruction.

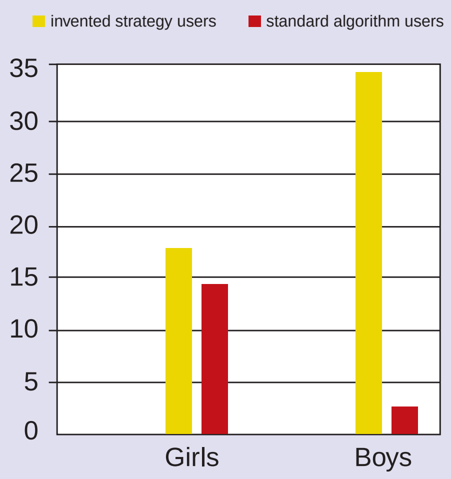
Girls' tendencies to use teacher-sanctioned strategies may persist throughout their schooling. In a study of high-ability high school students, males and females were similarly successful on a battery of SAT mathematics problems but differed in their approaches to the problems. The males attempted conceptual shortcuts and unconventional techniques, whereas the females tended to use school-taught, often less efficient techniques. This finding led the researchers to speculate that females were less adventurous in their problem solving and reluctant to apply their conceptual understanding to problems (Gallager and DeLisi 1994).

Cautious girls may prefer "one size fits all" approaches, which include standard algorithms and concrete strategies. Such approaches are usually accurate, easy to execute, and simple to explain. Very little risk is involved in choosing these strategies. When asked to explain such a technique, a child can rely on blocks to support the explanation. In fact, some children might say nothing and simply let the blocks illustrate their work. The standard algorithm offers a similar advantage: Because many people use it, explaining it is unnecessary. Girls often are interested in explicit communication and may be attracted to strategies that can be explained clearly and are familiar to other students in the room.

In choosing strategies that are most common in their classes and that are easy to explain, girls are making rational choices. Unfortunately, these choices seem to limit understanding. These girls do not try to use the invented strategies that lead to understanding. They may shy away from such strategies because they involve adjustments and must often be reinvented for each new problem with its unique pair of numbers. However, the act of adjusting seems to increase the power of mental mathematics and invented strategies as learning experiences because the child has to actively think through each new problem rather than go on automatic pilot.

FIGURE 1

Number of girls and boys using invented strategies and standard algorithms (32 girls and 37 boys participated in the study)



We must remember that not all girls are cautious in their approaches to problems. As mentioned before, half the girls in the longitudinal study (Fennema et al. 1998) used the same kinds of invented strategies or mental mathematics techniques used by the boys. Many girls do not need the interventions outlined below, but some will. Very few boys need this type of assistance, but none of the interventions will inhibit a child from developing more sophisticated strategies.

Teacher Interventions

The data from the longitudinal study indicate the danger in children's leaping directly from using concrete strategies to algorithms without generating their own abstract strategies. Children who do not use mental mathematics or invented strategies are less likely than those who do to develop conceptual understanding of how multidigit numbers "work." The data show that even in first grade, boys are more inclined than girls to attempt to "work in their heads." Although one response to this information about girls' overuse of manipulatives might be to eliminate blocks and other concrete objects from the curriculum, this reaction goes against both common sense and theoretical analyses that underscore the importance of using manipulatives to develop understanding.

These girls do not try to use the invented strategies that lead to understanding

If the elimination of manipulatives is not the answer, then teachers should explore additional means of promoting mental mathematics and invented strategies that build understanding. The following suggestions will encourage children to use mental mathematics while still allowing those at the concrete developmental level to work with manipulatives.

- Encourage the use of a variety of strategies at all times. Let children know that their efforts to work "in their heads" are valued and that, eventually, they all should begin to use abstract strategies.
- When a child solves a problem but cannot explain how she did so, do not prompt her to use manipulatives. Rather than ask her to show you what she did, support her in explaining her thinking. You might ask, "What number did you start with? What did you do next?" and so on.
- Encourage children to challenge themselves. Ask, "Can you think of another way to solve that problem? Can you try working in your head?"
- If a child uses manipulatives, ask her to explain what she did without giving her access to the blocks. In other words, push the blocks aside and say, "Now tell me what you did with the blocks." This practice will prompt the child to reflect on her actions. By imagining her actions, she will begin to develop mental pictures of the blocks, on which she might be able to operate in the future.
- Try to create a spirit of risk taking in mathematics. Encourage students to try new things. Applaud mistakes as opportunities for learning. Give all children the message that you do not expect them to be perfect.
- To foster the habit of trying mental-mathematics strategies, give children easy problems when they have no access to manipulatives. For example, while the children are lining up for recess, you might ask, "If we have 22 children in our room and 19 children in Mr. Smith's room, how many children will be on the playground at recess?" Invite the children to explain how they calculated their answers to help others adopt similar techniques.
- Keep a close eye on girls to "catch them in the act" of using mental mathematics. When they attempt such strategies, give them opportunities to verbalize and reflect on the strategies as they articulate them. Celebrating these attempts prompts children to try again in the future.

Teachers and researchers often wonder why many girls and a small number of boys latch on to concrete strategies and standard algorithms and fail to try other techniques. Various factors, many of which teachers cannot control, probably play a role. Teachers can be aware of the phenomenon, be alert to automatic-pilot syndrome, and do all they can to promote the reflection and invention that have been shown to generate understanding.

As teachers, we certainly do not want to insist that all children use manipulatives when many are capable of using more sophisticated strategies. We do not want to send the message that manipulatives are the favored problem-solving technique. Children can use manipulatives in the same unthinking ways that they use algorithms. Some girls seem particularly prone to this practice, and we need to push them to be more inventive and adventurous in their approaches.

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