Creating Numerical Scales for Measuring Tools

Sharon L. Young and Robbin O'Leary

Young children show an early interest in measuring. They want to know how tall they are and how much they weigh, and they want to measure the ingredients to make cookies. We wanted to tap this natural enthusiasm for measuring by involving students in activities that would encourage them to think mathematically about the need for creating measuring tools with numerical scales.

Many mathematical ideas must be addressed before students can understand measuring scales on rulers, measuring cups, and thermometers. These ideas include understanding the importance of repeating the same-sized unit to determine a measure; understanding that units of different sizes yield different numerical measures of the same object; and realizing the inefficiency of measuring with individual units, such as wooden rods that are each 1 centimeter long, as opposed to connected units, such as a centimeter ruler.

First-grade students in two schools, one in a large city and one in a rural and suburban area, participated in activities that addressed these ideas. The authors, one in each school, led the activities.

FIGURE 1

Measuring the tabletop with "king size" footprints

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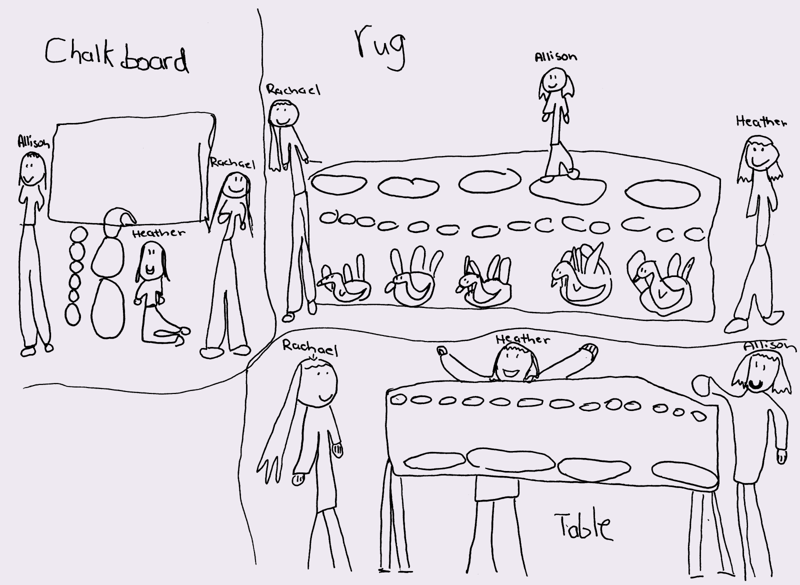
FIGURE 2

Student drawing of measuring activity using king and carpenter footprints



Developing a Numerical Scale for a Linear Ruler

Students should measure lengths using individual nonstandard units before they begin working with nonstandard rulers and numerical scales. The following beginning linear activities focus on measuring with individual nonstandard units.

Using individual nonstandard units

We used Rolf Myller's *How Big Is a Foot?* (1991) to kick off the measurement investigations. This delightful story tells about a king who asked an apprentice carpenter to build a bed that is six feet long and three feet wide. The carpenter measured with his own small feet rather than the king's large feet and, thus, made a bed that was too small. After hearing the story, the students worked in groups to explore what happens when different-sized units are used to measure the same object. Each group was given ten individual "king size" footprints and ten individual "carpenter size," shorter footprints to measure the lengths, widths, and heights of objects in the room. The students recorded the measures first using king-footprint units, then using carpenter-footprint units. Their task was to answer the question "What happens when we measure something with long footprints, then with short footprints?"

Several problem-solving situations arose during this activity. One group was overheard comparing its bookcase measurement of seven king units with another group's measurement of ten king units and wondering why the difference occurred. One budding mathematician exclaimed, "We must have done something wrong! Let's do it again and figure it out." As the group remeasured the bookcase, another student shouted, "I know what's wrong! The footprints can't be on top of each other." After a brief discussion, the group concluded that units had to be placed end to end with no spaces between them and no overlapping (see **fig. 1**).

In each classroom, one of the groups was quite bewildered while trying to measure an object that was longer than ten footprints. Both groups had already used their ten king footprints and tried to borrow extras from other groups, but the other groups were actively using their footprints and had none to lend. We directed the students to think of a creative way to use just the ten footprints that they had been given. Soon, in each group, a student suggested reusing footprints that had already been counted by placing them on the floor after the tenth footprint. Two other groups, one in each classroom, had already seen the possibilities in reusing footprints and used just two footprints, repeatedly laying one down, then the other, then moving the first, until they reached the desired length.

Next, all groups reported their measurement results for the same three objects (see **table 1**). The students noticed that the shorter footprints generated a larger number in the measurements than the longer footprints did. Most students did not understand this phenomenon until one student demonstrated the idea concretely while explaining it aloud. He measured the desk first with king footprints, then with carpenter footprints placed next to the king footprints. Seeing the units side by side

TABLE 1

Number of king footprints and carpenter footprints reported by four groups of students

	Door-Handle Height		Desk Length		Countertop Length	
	King	Carpenter	King	Carpenter	King	Carpenter
Group 1	4	6	6 1/2	11	12	22
Group 2	4	6	6	12	13 1/2	21 1/2
Group 3	4	6	6 1/2	11 1/2	13	15
Group 4	4	6	7	11	13	20

FIGURE 3

Making and using a king-footprint ruler

helped others understand that more carpenter footprints were required to measure an object because they were shorter. The students then drew pictures of the measurement activity to show this concept (see **fig. 2**).

One child asked why the measurements from all the groups were not the same, given that they had all measured the same objects using the same units. In the discussion that followed, the students talked about how close some of the measurements were—such as 22 versus 21 1/2 footprints—and why the measurements were not exactly the same. In doing so, the students addressed the notions of approximate measures and measurement error.

Making a nonstandard ruler

In the next lesson, the students discussed four difficulties that they encountered when measuring with their footprint units: (1) "Forgetting what number we counted to and having to count again"; (2) "Keeping the footprints straight, they go all crooked"; (3) "It was hard to keep them touching the one in front"; and (4) "When we measured things that go up, like the door and the chalkboard, the footprints fell down."

One solution to these difficulties was to tape the footprints heel to toe onto long strips of adding-machine tape. When one group incorporated numbers on their footprints, the idea caught on, and soon all the groups had created nonstandard footprint rulers with numerical scales (see **fig. 3**). The students wrote the numbers in the middle of the footprints, however, rather than at the end. Consequently, an object that measured 5 1/2 footprints long could mistakenly be recorded as 6 footprints long, as shown in **figure 4.** When the whole class discussed this problem, some students suggested writing the numbers on the adding-machine tape right where the toe of one footprint touched the heel of the next footprint. This idea was important for students to generate on their own.

The ruler made from adding-machine tape produced a new challenge: How could the ruler be used to measure something longer than itself? After much discussion, most groups worked out a solution by placing a pencil or other object at the end of the tenth footprint; then moving the entire ruler, placing the starting point next to the pencil; and continuing the measurement process.

FIGURE 4

An umbrella that is 5 1/2 footprints long can be inaccurately measured as 6 footprints long when the numbers are recorded in the middle of a footprint.

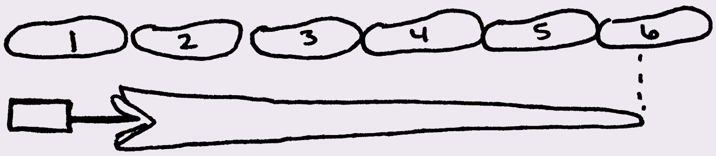


FIGURE 5

Containers A and C each measure four cups, container B measures three cups, and container D measures two cups.

Creating an inch ruler

The third activity focused on transferring the knowledge gained using nonstandard units and rulers to using standard units of an inch and a foot. Each student was given a one-inch-long piece of tagboard and asked to use it to measure a one-foot-long strip of tagboard. The students had difficulty measuring objects with this tiny unit and arrived at answers ranging from ten to thirteen inches. In a class discussion, the students were asked how measuring with inches could be made easier. One student suggested, "Get a bunch of them, and glue them down like we did with the footprints." When asked whether they could find a way to make a length of one-inch units using only the one unit that they each had, the students were initially silent. Eventually, a student asked, "Can't we just draw around the inch and keep sliding it and drawing it each time we slide it?" Using this method, most students got their marks to come out fairly accurately. Another interesting dilemma arose, however, when the students attempted to put numbers on their inch rulers: The written numbers went only to 11, but the tagboard was twelve inches long. The end of the twelfth inch was the edge of the foot-long strip and had no mark. Writing 12 near the end of the ruler helped students see that a standard foot has twelve inches.

Developing a Numerical Scale for a Capacity-Measuring Jar

The activities for measuring capacity involved the use of nonstandard units only. Again, the activities using individual nonstandard units preceded those for developing a measuring container with a numerical scale.

Using individual nonstandard units

The first capacity activity focused on measuring the quantity of water or rice that a large container would hold when small drinking cups were used as the unit of measure. Some students filled their containers with water or rice, then poured the substance into small drinking cups, counting as each cup was filled. Other students started with empty containers, then poured in small cups full of water, counting the number of cups required to fill the containers (see **fig. 5**). Students measured four different containers, then compared their capacities by comparing the number of small cups used to fill them.

After the students measured the containers, they discussed the difficulties that they had encountered, including spilling because of the small size of the cups, having difficulty filling the cups all the way to the top, and losing count of the number of cups when only one small cup was repeatedly used to fill the container. When the students were asked for solutions to solve these problems, most of their responses dealt with using larger cups, but they did not know how that solution would help them determine the number of small cups that the containers held. Although the students had recently made a numerical scale for a linear ruler, transferring that idea to capacity units was too difficult for them. This difficulty is partly caused by the problem of visualizing how individual cupfuls would appear inside the containers, given that cupfuls of water or rice flow together.

Making a nonstandard measuring jar

The students were asked what containers their families used at home to measure liquids. Many described a measuring cup with lines and numbers

FIGURE 6

Marking the water level on a measuring jar

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for liquid capacity. When asked what purpose the lines and numbers had, one student replied, "So if you want to just pour one cup of something, like juice, you just pour it all the way up to where it says '1 cup' but don't go over, or you get too much." Asked in what other situation they had used lines and numbers to help measure something, many students referred to a ruler, but they failed to see the connection with measuring capacity. Finally, one student inquired, "Do you mean if we tape our inch ruler to the side of the measuring cup?" The students concluded that the inch ruler would not work; instead, a unit that holds water was needed. They were challenged to create a measuring jar with lines and numbers to indicate numbers of small cups.

Each student group was given a large glass jar with a piece of masking tape attached to it from top to bottom. Through exploration, some students saw a connection between pouring a small cup of water into the larger container and marking on the tape to indicate the water level as each small cupful was poured in (see **fig. 6**). A few groups, however, poured several small cups of water into the containers before making that connection and having to start over.

Once work with the measuring jars was completed, two groups sitting near one another realized that the lines and marks on their measuring jars did not match. They rechecked their work, but both sets of marks appeared to be accurate. The water-level marks were closer together on the container with the greater capacity. Verbalizing the idea that a wider container could hold more than a narrower one eluded the students until one of them suggested that the wider jar held more because "it is fatter, and so the water squooshes down more." Another child suggested that the water had more room to flow into, so each small cup poured in was "flatter." The students in one classroom wrote their own instructions for making a measuring jar. One student's directions are shown in **figure 7.**

Developing a Numerical Scale for a Thermometer

Measuring temperature is different from measuring length or capacity, both of which can be measured directly with individual units, a ruler, or a marked container. Temperature must be measured using indirect means; specifically, instead of measuring temperature, we measure temperature's effect on the liquid contained in a thermometer. To measure temperature, the students started with unmarked thermometers to explore the ideas of measuring temperature through comparisons.

Using indirect comparisons

We asked the students, "How do we know when something is hot or cold?" They generally agreed that a person needs to touch something or get very close to it to find out whether it is hot or cold, although several students suggested using a thermometer. When asked whether touching an object was necessary to decide whether it was short or long or whether a container held a lot or a little, the students agreed that a person could just look at the object to decide rather than touch it. The students began to sense that measuring temperature would be different from measuring length or capacity.

We then gave students thermometers without scales. The calibrated scales on the thermometers' plastic holders were covered so that the marks and numbers were not visible. The students were asked to draw a mark on the tape alongside the glass tube to indicate the initial level of the liquid; this measurement was room temperature. Then the students recorded four more marks for the level of the liquid when the thermometer was placed (1) in hot water (be careful), (2) in ice water, (3) in ice water with salt added, and (4) under the arm to take body temperature.

This activity gave students experience with the effects of heat and cold and in making comparisons of those effects. The students could see that the solution of ice water with salt was colder than the ice water alone and were surprised to find that adding salt made the temperature of the water drop, although they could not feel this change by touch. They also observed that their bodies were warmer than the classroom air.

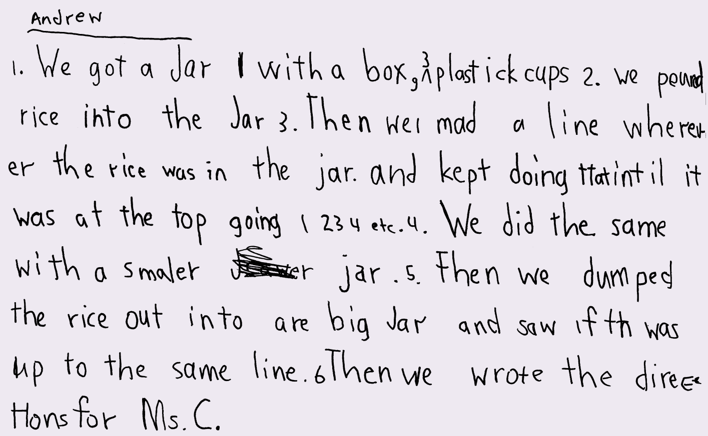
Creating a nonstandard numerical scale for a thermometer

In discussing the uses of temperature measurements, the students mentioned cooking, reporting on the weather, checking for illness, and reading thermostats in buildings. One student stated that these temperatures differed from the ones that they had recorded because theirs had no numbers. We asked, “Where do you think the numbers for temperatures come from?” but no one could answer. One student proposed to mark the lowest measurement with a 1, the next with a 2, and so on. All were satisfied until we asked whether some numbers could be close to one another and others, further apart. The students believed that the numbers should be equidistant but could not make the connection with using a nonstandard unit to establish the scale. Asked whether they could use footprints or small cups for the scale, the students laughed, recognizing the inappropriateness of these units. We finally suggested that the students use tiny colored dot stickers as nonstandard units to create their scales. The students then placed these dots in a vertical line along the side of the glass tube (see **fig. 8**). They numbered or counted the dots from the bottom of the thermometer. When asked why the smaller numbers were at the bottom, one student replied, “Because when the red [liquid] goes higher, the numbers should go higher.”

Although recording the temperature of ice water as “3 dots” and body temperature as “10 dots” may seem strange to adults, using this numerical scale of nonstandard units for measuring temperature made sense to these students. In fact, the dot scale probably made more sense than such abstract units as degrees Fahrenheit or degrees Celsius. Finally, we asked, “If you called the doctor about being sick, would she know what you meant by a temperature of fifteen dots?” This question elicited a discussion about the need for standard units for measuring temperature.

FIGURE 7

One student’s written description of the task of making a measuring jar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1. We got a Jar with a box, 3 plastick cups 2. we pourd rice into the Jar 3. Then wei mad a line wherever the rice was in the jar. and kept doing thatintil it was at the top going 1 2 3 4 etc. 4. We did the same with a smaler jar. 5. Then we dumped the rice out into are big Jar and saw if th was up to the same line. 6 Then we wrote the directions for Ms. C.

FIGURE 8

Student measuring with a dot thermometer

Summary

Throughout the activities, the students were encouraged to think like mathematicians. They explored length, capacity, and temperature at a deeper level than is typical of many measurement activities. We posed questions and encouraged the students to both ask questions and find ways of answering them. In the process of developing numerical scales, the students learned that the size of a unit affects the numerical values in a predictable fashion and must be selected in relation to the size and type of object being measured. They saw for themselves the inverse relationship between the size of the unit and the number of units needed to measure an object. Using nonstandard scales prompted the students to question “facts” that they might previously have accepted at face value. After completing these activities, the students’ ability to read standard scales was enhanced, as was their understanding of the need for such scales.

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