When Flash Cards Are Not Enough

"Why do you *always* make me tell you so many ways?" puffed eight-year-old Ashleigh. Then she laughed. "OK, OK, I guess I don't really mind when you ask me that. So here goes. . . . I know the answer for 9 + 7 is 16 because if I put 1 more with the 9 I'll have 10; then I have 1 less with the 7, so 10 + 6 is 16. I also know the answer is 16 because 8 and 8 make 16, and if I give 1 to 7 that will be 8, and now 9 has only 8 and that makes it like 8 and 8, which is one of my doubles that I already know. Hey, wait, I could use my doubles strategy because now I see another double! Seven plus 7 is 14, and 2 more from the 9 makes 16. . . ."

Ashleigh's confidence in her mathematical response is one of several changes that have occurred since we began working together. She also has increased her speed in solving simple addition and subtraction problems, applied a variety of strategies to find sums and differences, and exhibited substantially higher accuracy rates.

The Problem

My work with Ashleigh began when a neighbor found out that I like to "do mathematics" with elementary students. She asked me to work with her third-grade daughter, Ashleigh, who had "a problem with subtraction," she said. On timed tests of addition and even multiplication facts, Ashleigh was receiving grades of 90 to 100. On tests of subtraction facts, however, she was consistently coming home with scores in the 50s or lower. Ashleigh's mom said they had studied with flash cards many times, but Ashleigh still could not remember her subtraction facts. "Also, she does this thing with her fingers when she does her subtraction homework," Ashleigh's mom said. "Her fingers get to flying, counting up and down and backward. . . . You'll have to watch her to see what I mean."

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When I did watch Ashleigh, I realized that she was trying to use a prescribed touching technique and also count backward with her fingers to do subtraction. These techniques became complicated when she needed to solve a problem such as 17 – 8. Her use of fingers caused accuracy errors, her dependence on touching the numerals to subtract was slowing down her performance rate, and her backward counting sometimes became muddled and ended in wrong answers.

I noticed another side effect of her dependence on only these two techniques: She was not thinking about the numbers and their relationships. She was thinking only about counting backward or about where to touch the numerals. When she wrote her solution, she was unaware of whether it "made sense"; it simply was the answer she got when she was finished. Her lack of awareness of number relationships showed again when she was playing card games. Ashleigh knew that a 4 and a 6 make 10 and could get this answer when she added on paper. But when she tried to solve 10 – 6 or 10 – 4,

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she had no idea what the solution was.

Ashleigh's limited ability to explain her reasoning is important to note. When I asked her how she knew that 8 – 5 is 3, she responded, "I just do!" I asked her what strategies she might use to complete a set of subtraction problems and she said, "I'll keep on working." I prompted her to think about other strategies she would use. "I'll try my best," she said.

The Solution

My plan to increase Ashleigh's computational fluency involved three parts:

- She needed to develop automaticity for subtraction, as well as addition, facts.
- She needed to develop flexibility in thinking about numbers based on understanding the relationships among whole numbers and being able to decompose and recombine numbers easily, and she needed to be able to talk about these concepts.
- She needed more efficient strategies to help her quickly find differences, as well as sums.

Principles and Standards for School Mathematics (NCTM 2000) describes computational fluency for students in grades 3–5 as "having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently" (p. 152).

The approach that I used to meet Ashleigh's instructional needs could be implemented in any effective elementary mathematics classroom. It involves serious instruction embedded in the context of engaging activities. A unique characteristic of Ashleigh's situation is that she did not have peers to work with on the mathematics games. (I made some adaptations in the games so that adults could play with her and the practice still would be effective.) Each of our sessions was about fifty-five minutes long, and we met twice a week for about six weeks. Ashleigh was able to see results in her school performance after only three of the sessions.

A focus of my plan was to help Ashleigh "see" what she was doing when she solved problems, so that she would have several ways to work with the same problem and would be able to relate what she knew about one set of numbers to another set of numbers. Thinking about their own thinking—*metacognition*—is a fundamental component of children's reasoning processes (Tang and Ginsburg 1999).

Familiarity also was an important focus for Ashleigh. Researchers have noted that students become familiar with basic operations by using them often

in different contexts. After a while, they are able to remember many or most of the more simple calculations (Mokros, Russell, and Economopoulos 1995). The ultimate goal for Ashleigh is that she becomes so familiar with the number relationships of the basic number combinations that she knows them automatically (has them memorized) and can manipulate (decompose or recompose) the numbers. Her ability to do these things will not only help her learn addition and subtraction facts but also empower her as a mathematician, overall.

The "Class" Structure

Routine is essential to high productivity, so we followed the same series of activities each time we worked together. The sequence included the following activities: Warm-Up; Automaticity Check; Numbers in Context, using "story" problems; Strategy Instruction and Games, to practice using the strategies; and Plan for Practice at Home.

Warm-up

Each session began with a warm-up called "What Is It?" Ashleigh would roll two number cubes and shout out the sum of the upturned faces. She was not allowed to use her fingers for counting. She had to look at the number cubes and say the sum immediately. If she could not, I would say it for her and she would repeat it. This activity developed her skills in subitizing—recognizing a quantity without counting—and automaticity for addition facts to 12.

Automaticity check

Next, Ashleigh would complete a page of thirty basic addition and subtraction equations composed of numbers ranging from 1 to 19. I told her to work down the page, writing the answer to a problem as quickly as she could. If she did not immediately know the sum or difference, she would place a circle in the solution area. Later, she would return to the problems that she could not answer quickly and work out the solutions. Revisiting the problems is an important time for instruction. As she solved problems such as 16 – 8, I would ask her to describe what she knew about these numbers. I directed her attention to relationships that she did not immediately see, such as "A 'double' 8 equals 16" or "The inverse of 8 + 8 is 16 – 8."

After several sessions of practice, Ashleigh was consistently considering number relationships and ways to use them. For instance, when she became "stumped" on 17 – 9, she quickly said to herself, "Seventeen minus 10 is 7, so 17 – 9 must be 8. Hey, that's right because I know that 9 + 8 is 17." Feeling proud of her reasoning, she added, "I talked myself through it! You just have to look at it and arrange the numbers in your head. Then you can figure it out." Using a benchmark or "friendly" number to solve the problem and spontaneously checking the correctness of her solution were new skills for Ashleigh that emerged as she became more aware of her knowledge of numbers and her reasoning abilities.

When Ashleigh began to think aloud about her reasoning, I noticed that she was using techniques such as finding missing addends to solve problems such as 15 – 7. She said, "Well, I could think of what goes with 7 to equal 15. Well, I know that 8 does, so 15 – 7 must be 8."

Numbers in context

Studying numbers outside of real-world contexts often does not make sense to young learners, so I made it a point to have fun with at least one problem each session that was embedded in a story. After I told the story, Ashleigh first attempted to solve it mentally, then in writing. This practice further developed her reasoning skills and her ability to decide which strategy to use to solve the problem. Of course, it also sharpened her mental mathematics skills. The problems that we used were related to real-life situations and relevant to Ashleigh's life or the season. For example, one problem was the following: "Ashleigh is planning to have a party for Halloween. She invites 16 friends. Twelve of them respond that they are coming, but 4 later get sick. How many friends attended Ashleigh's party?" Several ways exist to arrive at the correct solution to this problem. We spent time discussing the different ways and thinking about which approach was the most efficient or "quickest."

FIGURE 1

Strategies for thinking about addition and subtraction

Doubles Plus One. Given a problem such as 5 + 6, look for a double number such as 5 + 5 and add 1. To solve the problem 8 + 7, use the double of 7 + 7 and add 1.

Ten Plus. Make a 10 as a "friendly" number, then work with the remaining numbers. For example, for the problem 4 + 7, think, "Three and 7 make 10, and 1 'left over' makes 11." For 17 – 8, think, "Seventeen minus 7 is 10, and taking away 1 more would be 9."

Neighbors. If a number appears next to another number on a number line representing the set of whole numbers, the distance between them is 1; therefore, 7 – 1 = 6 and 6 + 1 = 7.

Two Away. Visualize numbers sequentially and think of the number after the next one in the counting sequence. For example, 7 + 2 = is "two away" from the 7 in the greater direction, or 9. Likewise, 7 – 2 = is "two away" from 7 in the lesser direction, or 5.

Complements. Think of one set of numbers as "complementing," or "hanging around with," one another. For example, for the problem 6 – 4 =, think, "What goes with 4 to make 6?" In this case, the solution would be 2 + 4 = 6.

Strategy instruction and games

At this point in the lesson, we would talk about a strategy for solving problems, such as "doubles plus one," "ten plus," "neighbors," "two away," and so on (see **fig. 1**). Arthur Baroody and Dorothy Standifer (1993) provide detailed descriptions of addition and subtraction strategies for primary-grade students and cite interesting research studies about these strategies. After discussing a strategy, Ashleigh and I would play a game to practice and enhance the use of the strategy and to develop automaticity, the immediate knowledge of certain number combinations.

Sometimes when I visit classrooms, teachers tell me that they are using games to teach mathematics and that their students still do not know their facts. In order for students to learn from games, the teacher must help them focus on specific number concepts, notice strategies they are using, and talk about their discoveries. Talking about the mathematics they are doing gives students chances to clarify their thinking (Mokros, Russell, and Economopoulos 1995). When these strategies accompany the game playing, students become more proficient in their number sense.

The games that Ashleigh played came from a variety of resources and were selected to strengthen her skills in addition, mental mathematics, logical reasoning, and subtraction (see **fig. 2**). As she became more skillful, I made changes in the games so they were more challenging for her.

Practice at home

After each session, Ashleigh would add the new game we had learned to a games folder that she made out of construction paper. The folder provided a way to track how many times she played a game by herself or with a family member or friend.

She also took home a list of at least five equations written on a sticky note to put on the refrigerator. Ashleigh's mom was asked to use a specific procedure to help her practice these facts orally. If Ashleigh was unable to respond after a moment, her mother would tell her the correct answer, instead of letting her count with her fingers. This type of practice is similar to practicing sight words in reading, in which students do not sound out a word but recognize it as a whole unit. Because Ashleigh was working with the basic number combinations in other ways, the goal of this practice simply was to recognize the equation quickly.


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Summary and Final Thoughts

Developing computational fluency is a multifaceted task that underlies all further work with numbers. During effective instruction, students need experience with a variety of ways to solve problems, time to talk about their findings, and opportunities to apply some of their ideas so they can create their own understanding of how numbers work. Computational fluency will emerge as students use flexible strategies and demonstrate greater speed and accuracy.

The work with Ashleigh to help her learn subtraction "facts" was filled with surprises. Ashleigh was surprised that she could do so much with numbers and that she could know something about one number based on information she knew about another number. Ashleigh's mom was surprised by how quickly her daughter demonstrated results on timed tests at school. I was surprised that the things we did together—talking about number decomposition and re-composition and using materials such as number cubes, ten frames, and card games—were unfamiliar to Ashleigh. Her experience in learning about basic addition and subtraction facts had been limited to worksheets, timed tests, and flash cards. She is not alone; many students' computational experiences are short-changed in the interest of time (Mokros, Russell, and Economopoulos 1995). Teachers say, "If I do all that stuff, my students won't have time to memorize their facts."

Yet the opposite often is true. The sequence of activities that I used to help Ashleigh develop her number sense and improve her command of basic number facts is easily replicated in a classroom setting for whole-class practice or for a small remedial group. Students can be taught how to play the warm-up activity of "What Is It?" in pairs. "Automaticity Check" can be administered much like an old-fashioned spelling test; that is, the teacher says ten equations one at a time and the students write the answer, then check their own work against a list posted on the board or overhead projector that shows the equation and the answer. The teacher or students can create a story problem for "Numbers in Context" that relates to the students' interests or seasonal activities. After they solve the problem, students can talk in pairs or foursomes about the variety of ways that they arrived at the answer or the variety of answers that are possible for the problem. "Strategy Instruction and Games" can be demonstrated on an overhead projector. The teacher can briefly discuss a strategy such as "ten plus" or "two away," then model with a student partner how to play a mathematics game for that day. Once students know how to play a variety of games, the teacher will need only to demonstrate and discuss strategies. While students are playing the practice game, the teacher should circulate, listening for conversation that indicates development of number sense. At this point, the teacher can offer individual instruction as needed. Finally, all students can make a "Practice at Home" folder with a list of games that they know how to play and instructions for playing those games with friends and family at home. They also can choose one or two equations that they want to practice to achieve greater automaticity.

The sequence of activities described in this article does not need to be used all year long. It can be used for one or two weeks when students are studying or reviewing addition number facts, then again for a few days when students are learning subtraction or multiplication facts or indicate that they need more instruction. The mathematics games, however, are engaging enough that many students want to play them again and again. Teachers should keep the game materials accessible in the classroom so that students can continue to build their

FIGURE 2

Games that Ashleigh played

For a complete discussion on how to use these games, see the resources from which they were taken. The descriptions in these books include many useful mathematics tips.

Number Ladder. Each partner creates a "ladder" with a number from 1–10 written in mixed order on each rung. (The ladders do not have to be alike.) Use a number cube and a bean or small marker. Put the marker on the bottom rung, then roll the number cube and add that number to the number on the rung where the marker is placed. If the correct sum is given, the student can move to the next rung and continue. If a mistake is made, the student must go back to the bottom of the ladder. When one partner finishes or makes an error, the other partner takes a turn. One way to use this game for subtraction practice is to roll two number cubes, find the sum, then subtract the number on the ladder from that sum. *Adaptation for playing at home with an adult:* The adult purposely makes at least one error, and the child partner must try to catch it; otherwise, the adult most likely will make it to the top of his or her ladder first. (Kaye 1987)

Pyramid. Using the number cards in a deck of playing cards, set up a pyramid with one card at the top, two cards overlapping the bottom edge of that card, three cards overlapping the edges of the two cards, and so on, until there are six cards at the bottom of the pyramid. Pick up cards with number combinations that equal 10. Only cards that are fully uncovered can be used. At first, students use only two cards at a time that represent addends such as 3 and 7. Once students understand how to play, encourage them to use more flexible thinking and as many cards as necessary to make combinations such as $9 + 3 – 2$ or $2 \times 3 + 4$ for their solutions. (Kaye 1987)

Double War. Two players each have a deck of playing cards with the face cards removed. Each player turns up two cards at a time. The players add the values of their cards to see who has the greatest sum. That person wins and takes all four cards. If the sums match, each player deals out three cards from his or her pile and then turns the top or third card over for a playoff. Whoever has the greatest number from the two cards turned up during the playoff wins all the cards from that play. *Adaptation: Double War/Difference:* Use the same procedure as Double War, but this time subtract the value of the cards turned up. The least difference wins. (This is simply an adaptation of the game of War played by children with a single deck of cards when the players decide which of the upturned cards is greater to win that round. Double War also can be played with products to practice multiplication facts.)

Five Makes Ten. Use a deck of playing cards with the face cards and tens removed. Deal each player five cards. Each player tries to make equations that equal 10. The player who makes the most equations wins the round. Recording the equations is a good idea. Players can use each card only once in the same equation. *Adaptation: "Six Makes Ten":* In this game, deal six cards to each player and follow the rules for Five Makes Ten. Notice how one more card increases the number of equation possibilities. (Kaye 1987)

Can You Make a Difference? Make a vertical list of numbers from 1 to 9. Using Ten Frame or Ten Grid cards, deal each player five cards and place them faceup. If a player can make an equation that has a difference from 1 through 9, he or she puts a check mark beside that number on his or her list. The player with the most checks wins. *Note:* Ten Frame or Ten Grid cards can be bought commercially, or they can be made by drawing a two-by-five array and placing dots inside the boxes to equal various numbers. For example, to represent the number 7, five of the boxes on one side would have a dot in them and two boxes on the other side would have a dot. The three remaining boxes would be blank. (Kanter, Gillespie, and Clark 1998)

The Game of Ten. Use a set of playing cards without the face cards. Place the deck facedown. The dealer picks up the top card and places it on the table faceup. The next player turns up the next card. If a ten is uncovered or can be made by combining the card with any uncovered, previously picked card(s), the player announces how he or she knows it is a ten and takes the cards used in that turn. If a player cannot make a ten, the card is placed faceup on top of the previous card and the turn goes to the next player. Cards can be used as they are uncovered. Continue until all the cards are used. (Baroody 1993)

Take 'em Out, Put 'em Back. Students play in pairs. Each pair needs one small paper bag labeled with the number of interlocking cubes placed (loose) inside. Note that the number of cubes inside the bag depends on the set of basic number facts the students are working with at the time. The first player takes out a handful of cubes, shows them to his or her partner, and asks how many are still in the bag. The partner answers, then checks the cubes in the bag. The partners return the cubes to the bag, switch roles, and play again. (Robertson et al. 1999) number concepts by playing the games before school begins, during indoor recess, or when they finish a class assignment early.

Every mathematics classroom is pressed for time to have students learn their basic facts and to accomplish many other curricular requirements. Yet Ashleigh is an excellent example of the rapid rate at which students acquire and apply information about numbers when instruction and practice are paired with concrete materials, engaging tasks, and time to reason and reflect.

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