

Models and images for understanding multiplication and division



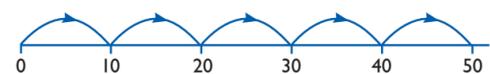
$2 + 2 + 2 + 2 + 2 = 10$
 $2 \times 5 = 10$
 2 multiplied by 5
 5 pairs
 5 hops of 2



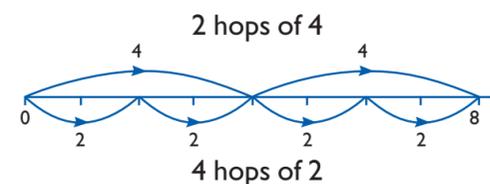
$5 + 5 + 5 + 5 + 5 + 5 = 30$
 $5 \times 6 = 30$
 5 multiplied by 6
 6 groups of 5
 6 hops of 5



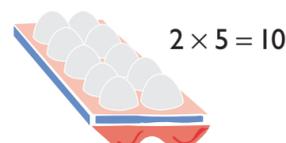
$10p + 10p + 10p + 10p + 10p = 50p$
 $10p \times 5 = 50p$
 5 hops of 10



$4 \times 2 = 8$
 $2 \times 4 = 8$
 $2 \times 4 = 8$
 $4 \times 2 = 8$

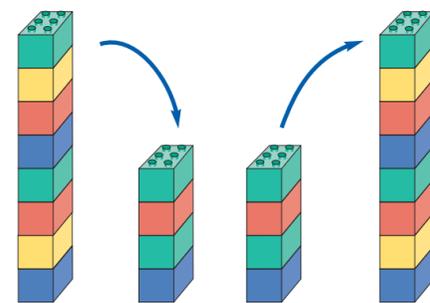
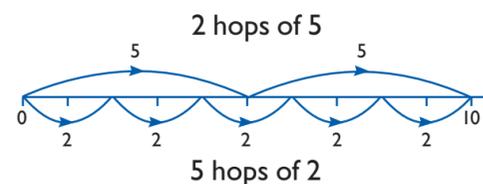


$5 \times 2 = 10$

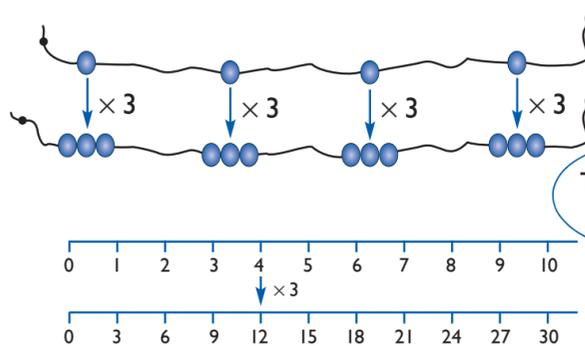


$2 \times 5 = 10$

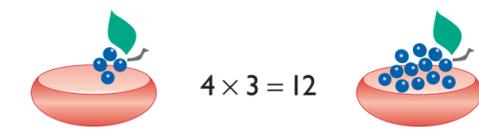
$5 \times 2 = 10$



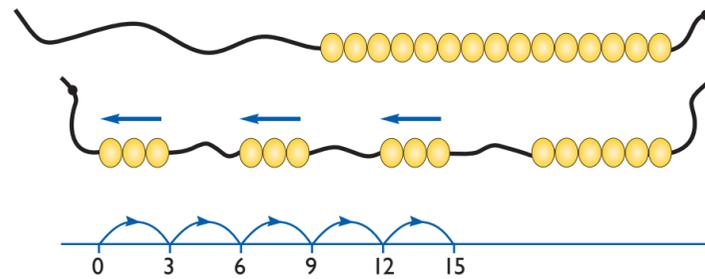
half of 8 is 4
 $8 \div 2 = 4$
 double 4 is 8
 $4 \times 2 = 8$



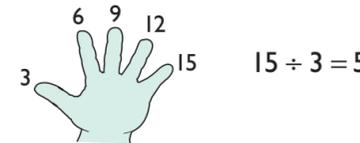
Three times as many



$4 \times 3 = 12$



How many 3s in 15?



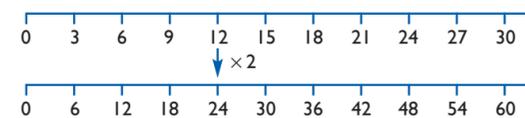
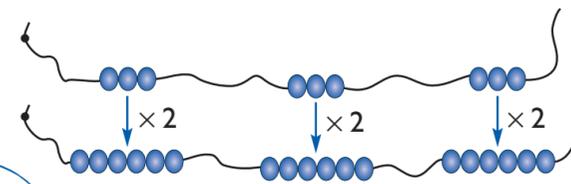
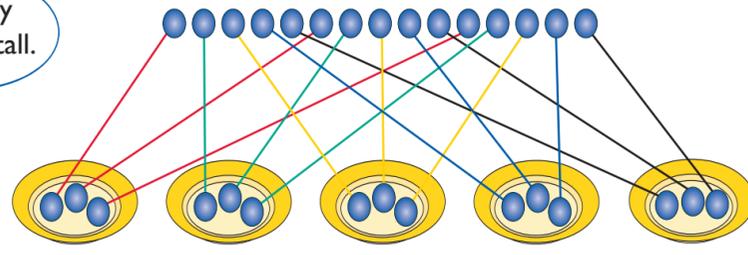
$15 \div 3 = 5$



5 hops in 15. How big is each hop?

$15 \div 5 = 3$

15 shared between 5



Twice as many

$12 \times 2 = 24$

Progression

Reception

- Count in twos from 0 to 10, will we say 9?
- Count round the circle of children in twos, who do you think will say 60?

Year 1

Autumn

- Count in 10s from 0 to 100. Count in 10s to 50, holding up a finger for each 10 we count. How many 10s did we count?
- How many 10p coins are here? How much money is that?
- What is double 2?
- How many socks in 2 pairs?
- Share 6 biscuits between two people so each person has the same number each. How many do they have each?

Spring

- Count in 2s to 30. Count in 2s to 10, holding up a finger for each 2 we count. How many 2s have we counted?
- Count in 5s to 15. How many 5s have we counted?
- How many toes are there on two feet?
- How many gloves in 3 pairs?
- This domino is a double 4. How many spots does it have?

Summer

- Count in 3s from 0 to 30. How many 3s did we count?
- How many sides are there on 4 triangles? Count them in 3s.
- What is double 5?
- What is half of 10?
- I roll double 3. What's my score?
- There are 8 raisins. Take half, how many have you taken?

Year 2

Autumn

- What is $5 + 5 + 5$? How can we write this as a multiplication sentence?
- What numbers go in the boxes?
 $6 \times 2 = \square$; $\square \times 10 = 40$;
 $\square \times \triangle = 12$
- How many 2s make 8?
- What numbers go in the boxes?
 $6 \div 2 = \square$; $20 \div \square = 2$;
 $\square \div 10 = 3$
- There are 12 cubes. Make 3 towers the same height. How tall is each tower?

Spring

- What multiplication sentences could we write about this array?

- What are two 5s? Double 6? 6 times 2? 6 multiplied by 2? What is half of 12?
- If 6 grapes are shared equally between two people, how many will they get each?
- If Sarah counts in 2s, and Nigel counts in 5s, when will they reach the same number?
- How many 10s can you subtract from 60?

Summer

- If double 11 is 22, what is half of 22?
- If $12 \times 2 = 24$, what is $24 \div 2$?
- A giant is twice as tall as a 10m high house. How tall is the giant?
- One snake is half the length of another snake which is 20 cm long. How long is the shorter snake?
- I doubled 3, then doubled the answer. What number did I get? I halved this number and then halved it again. What was my answer?

Year 3

Autumn

- What is 10 added together 4 times? How can we write this as an addition? And as a multiplication?
- Describe this array.

- I have 4 stickers. If you had 3 times as many stickers how many would you have?
- What is the product of 6 and 2? Of 2 and 6? What is 5 multiplied by 3?
- Is 35 a multiple of 5?
- What numbers go in the boxes?
 $4 \times 4 = \square$; $10 \times \square = 80$; $\square \times 5 = 30$;
 $6 \times 20 = \square$; $\square \times \triangle = 60$

Spring

- If $7 \times 5 = 35$, what is $35 \div 5$?
- What four multiplication and division sentences can you write to using 2, 5 and 10?
- Share 18 between 2. Divide 25 by 5. How many fives make 45? How many 5p coins do you get for 35p?
- How many 5s make 35?
- Share 18 between 2. Divide 25 by 5. How many lengths of 10m can you cut from 80m of rope?
- A baker bakes 24 buns. She puts 6 buns in every packet. How many packets can she fill?
- What numbers go in the boxes?
 $16 \div 2 = \square$; $30 \div \square = 6$; $\square \div 5 = 7$

Summer

- How many lengths of 10 cm can you cut from 81 cm of tape? How many will be left?
- If you put 25 eggs in boxes of 10, how many boxes would you fill? How many eggs would be left over?
- Count a handful of pasta pieces by grouping them in 5s. How many 5s were there? How many left over? How many pasta pieces altogether? What division sentence could you write?
- What numbers go in the boxes?
 $17 = 5 \times 3 + \square$; $\square = 2 \times 8 + 1$
 $29 = 5 \times \square + 4$

Year 4

- What is the product of 15 and 6?
- If you add 10×5 and 8×5 , what multiple of 5 do you get?
- Is 40 a multiple of 5? How do you know?
- What numbers go in the boxes?
 $4 \times \square + 8 = 24$; $33 = 4 \times \square + \triangle$

Understanding multiplication and division

Potential difficulties

Children may:

- be able to double four by pairing two groups of four and counting the group of eight but do not associate this addition with the equivalent multiplication four multiplied by two;
- halve by sharing or forming pairs and counting but may not associate it with division by two or division between two;
- halve and double independently, without recognising they are inverse operations, for example, knowing that half of eight is four means that double four is eight;
- interpret $12 \div 3$ as 12 shared between 3 and use objects or pictures to share out the 12, but lose track of their recording as the numbers increase as they have no other strategy available such as counting in steps or groups;
- interpret 6×2 as six lots of 2 and apply repeated addition of two, rather than doubling the six;
- not be proficient in counting forwards and backwards in equal steps, and so make mistakes when carrying out repeated addition or repeated subtraction;
- understand multiplication as repeated addition and resort to this method without using known facts, for example writing 6×7 as $7 + 7 + 7 + 7 + 7 + 7$ and counting in 7s despite knowing that $5 \times 7 = 35$;
- understand division as repeated subtraction and use a method of counting back but lose track of the number of steps to the answer;
- carry out division by sharing or grouping but cannot cope with a remainder and do not recognise that a remainder must always be less than the divisor;
- associate \times with multiplication and \div with division and do calculations 8×2 and $16 \div 2$ but are not able to find missing numbers in statements such as $6 \times \square = 12$ and $\square \div 5 = 3$;
- recognise what calculation to do when word problems include the words times or share, but are less confident when other language is used such as product, divided by, remainder, and mistakenly associate 'how many?' and 'how much?' with addition or subtraction;
- understand multiplication as repeated addition, and division as repeated subtraction, but not as scaling up and down to prepare the way for later work in measures and on ratio.

Other useful models and images

Examples of progression and application in Years 4 to 6

These examples are drawn from section 6 of the *Framework for teaching mathematics from Reception to Year 6*.

Year 4

- Understand and use when appropriate the principles (but not the names) of the commutative, associative and distributive laws as they apply to multiplication.

$$\begin{aligned}8 \times 15 &= 15 \times 8; \\6 \times 15 &= 6 \times (5 \times 3) \\&= (6 \times 5) \times 3 = 30 \times 3 = 90; \\18 \times 5 &= (10 + 8) \times 5 = (10 \times 5) + (8 \times 5) \\&= 50 + 40 = 90\end{aligned}$$

- Understand that:

$86 + 86 + 86$ is equivalent to 86×3 or 3×86 ; multiplication by one leaves a number unchanged and that multiplication by 0 results in zero.

- Understand the operation of division either as sharing equally or as grouping and that division by 1 leaves a number unchanged.

- Understand that multiplication is the inverse of division (multiplication reverses division and vice-versa) and use this to check results.

- Demonstrate understanding of multiplying or dividing a whole number by 10.

- Respond rapidly to oral or written questions, explaining the strategy used.

Two eights; double 16; 9 multiplied by 3; divide 69 by 3. Is 40 a multiple of 5? How do you know? Is 72 divisible by 3? How do you know? What is the product of 15 and 6? Tell me two numbers with a quotient of 5.

Use known facts to answer:

$$\begin{aligned}7 \times 2 &= \square; 10 \times \square = 80; \square \times 5 = 35; \\36 \div 4 &= \square; 60 \div \square = 6; \square \div 3 = 7\end{aligned}$$

Use mental or mental with jottings to answer:

$$\begin{aligned}90 \times 6 &= \square; 8 \times \square = 560; \\ \square \times 90 &= 270; 320 \div 4 = \square; \\240 \div \square &= 60; \square \div 30 = 8\end{aligned}$$

- Give a remainder as a whole number.

$$41 \div 4 = 10 \text{ remainder } 1$$

- Make sensible decisions about rounding up or down after division.

I have £62. Tickets cost £8 each. $62 \div 8 = 7$ remainder 6. I can only buy 7 tickets.

Year 5

- Understand that, with positive whole numbers, multiplying makes a number larger, and division makes a number smaller.

- Demonstrate understanding of multiplying or dividing a whole number by 10 or 100.

- Understand that division is non-commutative.

$$72 \div 9 \text{ is not the same as } 9 \div 72$$

- Understand that a number cannot be divided by zero.

- Start to use brackets: know that they determine the order of operations, and that their contents are worked out first.

$$\begin{aligned}3 + (6 \times 5) &= 33, \text{ where as} \\(3 + 6) \times 5 &= 45\end{aligned}$$

- Respond rapidly to oral or written questions, explaining the strategy used.

Double 32; multiply 31 by 8; divide 56 by 7; what is the remainder when 74 is divided by 8?

What is the product of 25 and 4? What are the factors of 36? Is 81 a multiple of 3? Is 156 divisible by 6? How do you know?

Use mental or mental with jottings to answer:

$$\begin{aligned}80 \times 9 &= \square; \square \times 9 = 0.36; \\172 \div 4 &= \square; \frac{54}{\square} = 18\end{aligned}$$

Use written methods or a calculator to answer:

$$\begin{aligned}134 \times 46 &= \square; 2.7 \times \square = \square; \\(14 \times 60) + \square &= 900; 900 \div 36 = \square; \\(125 \div \square) + 2 &= 27; \frac{\square}{28} = 46\end{aligned}$$

- Begin to give a quotient as a fraction or decimal fraction.

$$43 \div 9 = 4\frac{7}{9}; 61 \div 4 = 15.25$$

- Relate division and fractions.

$\frac{1}{3}$ of 24 is equivalent to $24 \div 3$ or $\frac{24}{3}$ or 8.

- Solve simple problems using ideas of ratio and proportion.

Zara uses 3 tomatoes for every $\frac{1}{2}$ litre of sauce. How much sauce can she make from 15 tomatoes?

Year 6

- Demonstrate understanding of multiplying or dividing a whole number by 10, 100 or 1000.

- Respond rapidly to oral or written questions, explaining the strategy used.

Double 75; multiply 25 by 8; divide 15 into 225; what is the remainder when 104 is divided by 12? What is the product of 125 and 4? What are the factors of 98?

Use mental or mental with jottings to answer:

$$\begin{aligned}0.7 \times 20 &= \square; 0.3 \times \square = 2.4; \\172 \div 4 &= \square; \frac{\square}{25} = 39\end{aligned}$$

Use written methods or a calculator to answer:

$$\begin{aligned}738 \times 639 &= 13; (41 \times 76) + \square = 4000; \\4123 \div 365; (100 \div \square) + 5 &= 7.5\end{aligned}$$

- Give a quotient as a decimal fraction, rounding where appropriate to one decimal place.

$$676 \div 8 = 84.5; 85 \div 7 = 12.1 \text{ to one decimal place.}$$

- Solve simple problems involving ratio and proportion.

At the gym club there are 2 boys for every 3 girls. There are 30 children at the club. How many boys are there?