Investigations

Weighing the Options

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This department recognizes the importance of children's exploring hands-on and minds-on mathematics and presents teachers with open-ended explorations to enhance mathematics instruction. These tasks invoke problem solving and reasoning, require communication skills, and connect various mathematical concepts and principles.

A mathematical investigation—

- has multidimensional content;
- is open-ended and has several acceptable solutions;
- is an exploration requiring a full period or longer to complete;
- is centered on a theme or event; and
- is often embedded in a focus or driving question.

A mathematical investigation involves processes that include—

- researching outside sources;
- collecting data;
- collaborating with peers; and
- using multiple strategies to reach conclusions.

Investigations are somewhat structured in their sequence of activities. They come alive, however, through students' problem-solving decisions and strategies. Although having students follow a scripted sequence and set of directions for an investigation is possible, the NCTM Standards encourage teachers and students to explore multiple approaches and representations in their mathematical activities. As a result of their exploration, students also will use their reasoning and proof skills as they evaluate their strategies. The use of multiple approaches helps students find new ways of looking at things and understand different ways of thinking about a problem.

Introduction

The NCTM Standards discuss the importance of gathering data for analysis, but teachers also need to provide opportunities for students to investigate the rich data that already exist. The purpose of this investigation is to engage students in real-life, open-ended data analysis. This investigation allows the student investigators to analyze existing data along with data that they have collected.

The investigation originally was designed for and field-tested with students in grades 4–6. Students in grades K–3 may wish to investigate some of the estimating and measuring activities as well.

Materials

The student investigators will need—

- a tape measure (or string and a ruler);
- a bathroom scale;
- shipping boxes of various sizes, filled with items so that they are different weights;
- a 30-by-24-by-6-inch box (for Part II);
- pencil and paper for recording data;
- a calculator;
- access to the Internet or print-outs of shipping data;
- a "Gathering Data" reproducible sheet; and
- a "What Happens If . . . ?" reproducible sheet.

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Edited by **Alice Merz,** *amerz@isu.edu, Idaho State University, Pocatello, ID 83209-8059. This section is designed for teachers who wish to give students new insights into familiar topics in grades K–6. This material can be reproduced by classroom teachers for use with their own students without requesting permission from the National Council of Teachers of Mathematics. Readers are encouraged to send manuscripts appropriate for this section to the editor.*

Preparing for the Mathematics in This Investigation

To prepare your students for this investigation, it is important that you review and practice measuring the length, width, and height of a box and reading tables of data. The review also should include strategies for obtaining accurate measurements.

Investigation Question

How much does it cost to ship a package?

Setting the Stage

You are a business owner who needs to send a large amount of merchandise to your customers. What you spend on shipping the merchandise affects how much money your business makes, so you need to check out various shipping costs.

Part I: Gathering Data

Students need to collect information to help them analyze shipping costs. First, they need to gather information about the size of their box. Some shipping companies base their rates on weight, whereas others take the dimensions of the box into account. Then students need to gather information about the location they will ship the box from (its origination) and the location they will ship the box to (its destination). Finally, they need to gather information about the services that a shipping company offers and how the company calculates the cost of shipping a package.

Determining the size of a box

Tell the students to discuss and develop benchmarks that will facilitate reasonable estimates. Having materials on hand that match certain benchmarks, particularly for weight (such as a one-pound canned good or a five-pound bag of flour), can be helpful. Discuss when estimating the size and weight of a box might be useful or necessary.

1. Estimate the length, width, height, and weight of a shipping box. Record your estimates on the reproducible.
2. Measure the same box by finding its length, width, height, and weight and record your measurements.

Identifying the shipping destination

Choose a city in the United States to which you will ship the box (its destination). Each student should assume that the box's origination is the city

FIGURE 1

Cost table for shipping from Seattle

Destination	Shipping Company	Type of Service	Weight (in pounds)	Cost
Denver, CO	FedEx	2nd day	10	$24.25
Denver, CO	FedEx	Ground	10	$ 6.04
Burlington, VT	FedEx	2nd day	10	$26.75

in which the student lives, or at least is in close proximity. Use the following Web site to find the origination and destination zip codes: www.usps.com/ncsc/lookups/lookup_ctystzip.html.

Collecting services and rate information

The student investigators will gather shipping information from one of the shipping companies. This investigation uses FedEx, but similar information can be gathered from the United Parcel Service (UPS), located at www.ups.com, and the U.S. Postal Service (USPS), located at www.usps.com/shipping/ship.htm. Comparing shipping costs among different companies can be interesting and mathematically challenging.

1. In order to determine what kind of shipping service students should select (for example, overnight, second-day, or ground), decide how quickly the package needs to arrive at its destination. The following Web site provides a brief description of available shipping services: www.fedex.com/us/services/waystoship.
2. Use the following Web sites to enter the origin and destination zip codes and retrieve an appropriate zone and rate chart: www.fedex.com/us/rates/zone.html or www.fedex.com/us/rates/downloads (downloadable version).
3. Use the chart to find the cost of shipping the package by matching the weight of the package with the desired type of service. See **figure 1** for the costs of shipping a box from Seattle.

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Part II: What Happens If . . . ?

One interesting thing that can happen while doing data analysis is that investigators can encounter additional questions. For example, investigators might think that they have determined the shipping cost when they really have not taken everything necessary into consideration.

Students will use the Part II reproducible to investigate questions such as the following:

- Does it cost the same amount to ship one large box as it does to ship two smaller boxes of the same total weight?
- What methods of rounding do shipping companies use?
- Are any fees added to the shipping costs?
- Are there any special considerations for shipping an oversize box?

Students also are encouraged to consider additional shipping questions such as the following:

- Does a shipping company charge the same to ship a package to a nearby destination as it does to somewhere far away? (For example, would it cost the same to send something 2,000 miles away and 50 miles away?) Is that always true?
- How much does the package cost per pound in each zone? What do you notice about the cost per pound? ▲

Part I: Gathering Data

Name _____

1. Record your estimates and the actual measurements of a box used for shipping.

	Estimate	Actual
Length		
Width		
Height		
Weight		

2. Assume that you will mail the box from the city or town where you live. Decide where you want to ship your box, or the destination. Use the following Web site to locate the U.S. zip codes: www.usps.com /ncsc/lookups/lookup_ctystzip.html.

   Box's place of origin: _____ Zip code _____
   Box's destination: _____ Zip code _____

3. Use the following Web site to find out about available shipping options: www.fedex.com/us/services /waystoship. Record the shipping option that you will investigate: _____

4. Use the rate information from the following Web sites to find the cost of shipping your package: www.fedex.com/us/rates/zone.html or www.fedex.com/us /rates/downloads (downloadable version). Locate the weight of your package on the chart. Record and compare the costs of two other types of services for shipping your package.

Your Cost

Destination	Shipping Company	Type of Service	Weight (in pounds)	Cost

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Part II: What Happens If...?

One box or two boxes. Use data from the rate table at www.fedex.com/us/rates/zone.html to support your answers to the questions below.

1. Is it less expensive to send one heavy 20-pound box or two lighter boxes that total 20 pounds? ____
   _____

2. Is it always cheaper to package and send the boxes in the way that you chose above? Explain. ____
   _____

Rounding the weight of a box.

1. Why do you think that shipping companies round weights up to the nearest pound to determine the shipping cost? For example, an item weighing 5 pounds and 1 ounce is rounded to 6 pounds. _____
   _____

2. Can you suggest a different way to round the weight of a package that ensures ease of use and fairness? _____
   _____

Fuel fee. Some shipping companies add extra fees to their shipping costs. For example, some companies add fuel fees because of the increase in gasoline prices. However, the rate tables do not include these fees.

1. What would be the real cost for all the types of service that you listed in your cost table in Part I if the fuel fee is an extra $2.00 per package? Record your answers in the table below.

2. What would be the shipping cost if a 10 percent fuel fee were added to the costs listed in the table? (This would be the same as charging an extra dime for every dollar you spend.)

Revised Cost Table

Destination	Shipping Company	Type of Service	Weight (in pounds)	Cost	Cost + $2.00 fee	Cost + 10% fee
Denver	FedEx	2nd Day	10	$24.25	$26.25	$26.68

3. Does it cost more to pay a set fee or a percentage? Explain. _____
   _____

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Oversize boxes. Shipping companies do not treat all large boxes the same. Some large boxes are considered "oversize" boxes and are shipped at a higher rate because they take up more space than is allotted for a normal-sized box.

In order to find out if a box is oversize, determine the box's girth and weight. Each shipping company uses one of the following definitions of girth. Exploring the definitions with an actual box may be helpful.

Girth Definitions

1. Girth is the distance around the package at its widest point perpendicular to the length.
2. Girth is twice the height plus twice the width.
3. Girth is the distance around the package's thickest part.

1. Use the first definition to determine the girth of a box measuring 30 by 24 by 6 inches. Make a line that is perpendicular to the length. (The longest dimension is considered the length.) Put your measuring device along that perpendicular line to measure all the way around the box. What is the girth? _____

   _____

2. Using the second definition, what is the girth of a box measuring 30 by 24 by 6 inches? _____
   How did you determine which measurements were the height and width? _____

   _____

3. Using the third definition, what is the girth of a box measuring 30 by 24 by 6 inches? _____

4. Was the girth measurement the same using all three definitions? _____ Explain. _____

   _____

Oversize Definitions

1. A package is oversize if its combined length and girth is greater than 84 inches but less than 108 inches and it weighs less than 30 pounds. If it is oversize, the package will be charged at the 30-pound rate.
2. A package is oversize if its combined length and girth is greater than 108 inches and it weighs less than 70 pounds. It will be charged at the 70-pound rate.

5. Using the oversize definitions above, is the box that measures 30 by 24 by 6 inches oversize if it weighs 10 pounds? _____

6. Explain how the shipping company might come up with a cost of $36.80 for shipping a 10-pound box that normally would cost $14.00. _____

   _____

7. Based on your findings above and in the "One or two boxes" section, what can you conclude about whether using large boxes is better? _____

   _____

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