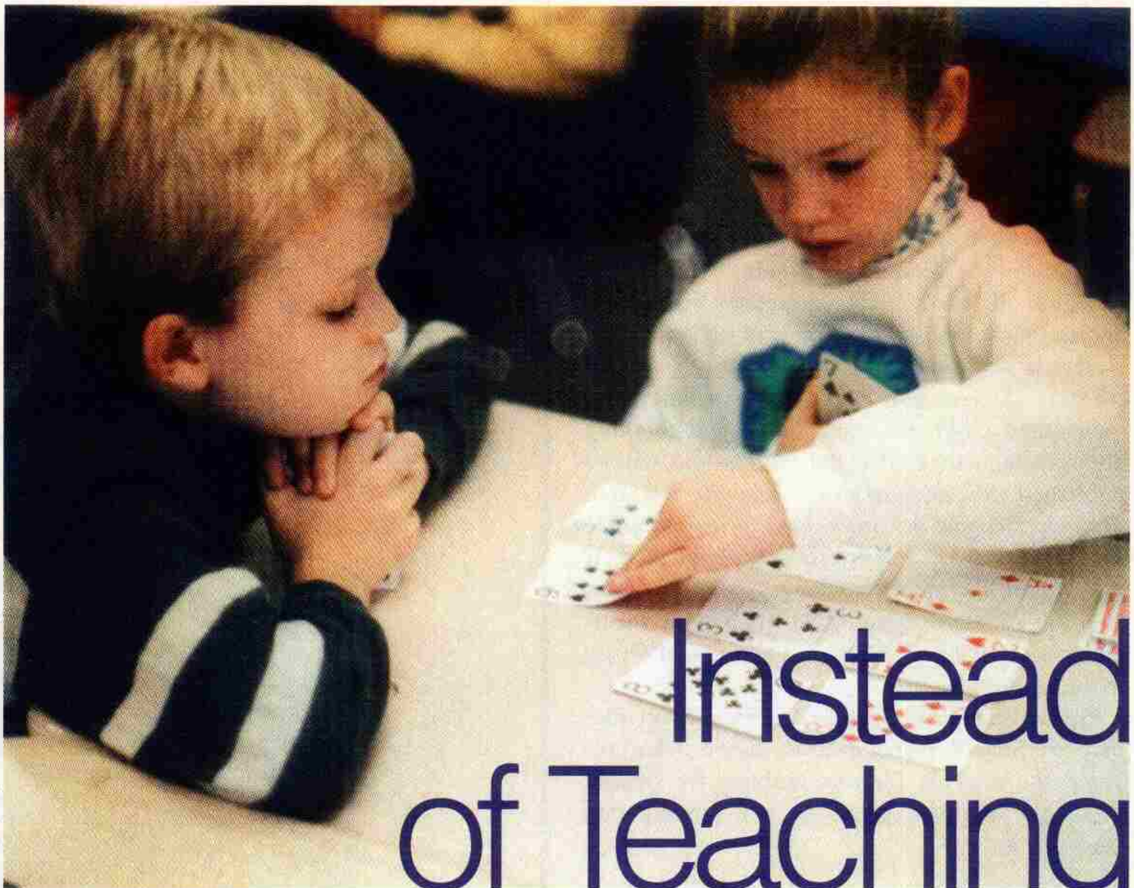

Instead of Teaching Missing Addends

Constance Kamii, Barbara A. Lewis, and Bobbye M. Booker

Recently, a principal wrote, "While visiting a first grade classroom one morning, I observed a lesson on missing addends. . . . Children were struggling with what they were being asked to do and the teacher was visibly frustrated. Afterward, when I discussed the lesson with the teacher, . . . she replied without hesitation, 'Most first graders can't do missing addends, but it's on the test' " (Morgan-Worsham 1990, 64).

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The purpose of this article is to present evidence showing that if children's numerical reasoning is strong, then formal instruction of missing addends is unnecessary. We first present data on how well five classes of first graders did without any formal instruction. We then explain the findings in light of Piaget's constructivism and discuss educational implications.

The children who participated in the study were 110 students in five classes of first graders attending two public schools. The schools were located in a suburb of Birmingham, Alabama, and the first-grade teachers involved were all constructivist in their approach to teaching. Instead of using a textbook or workbook, these teachers used situations in daily living, debate about how to solve word problems, and the kinds of games described by Kamii (1985). None of the five teachers formally taught missing addends, such as "4 + __ = 6."

In May, toward the end of a school year, the five teachers were asked to give a group test to their first-grade classes (shown in **fig. 1**). As one of us

FIGURE 1

The group test

3 + □ = 6
1 + □ = 5
4 + □ = 6
□ + 1 = 8
□ + 3 = 5
□ + 2 = 4

watched, each teacher distributed the test to the children in her class and asked, "What number do you think should go in the first box?" The entire class reacted with such expressions as "These are too easy!" The teacher called on one of the volunteers, who explained why "3" was the answer. The teacher went on to say, "Let's work one more problem together, to make sure you know what to do with the rest of the sheet." A volunteer quickly explained why a 4 had to go in the next box, and the five classes spent about four to seven minutes for the entire six-item test.

Ninety-two percent of the first graders handed in papers that had either no errors (85%) or only one error (7%). The five classes produced similar percents, and the first graders thus demonstrated their ability to solve missing-addend problems without any formal instruction in missing addends.

Eight of the 110 children (7%) demonstrated their difficulty with missing addends either by leaving the boxes empty or by writing in what appeared to be random numbers. We hypothesized that if these children did not receive any instruction in missing addends in second grade either, their thinking would advance to a level of being able to answer these questions.

In second grade in September, we attempted to follow the eight children who had demonstrated difficulty, but four had moved away. The teachers of the remaining four children were asked if, and when, they planned to teach missing addends. The same six problems were given to the four children in February and March, before their teachers formally discussed missing addends, and all four produced perfect papers.

Why Were These Problems Easy?

The children who took part in the study were not given any worksheets in kindergarten and first grade. Instead, they played many mathematics games every day and debated, for example, how many children are absent if twenty-one are present, whether it is necessary to count the "no" votes if thirteen students have voted "yes," and how a word problem can be solved in different ways. The numerical reasoning of children in constructivist classes has repeatedly been found to be strong (Kamii 1985, 1989, 1994). The strength of this reasoning and the children's reversibility of thought development explain the ease with which these first and second graders solved the missing-addend problems.

Games involving "4 + ? = 5" and "4 + ? = 10"

One of the many games the children played in kindergarten was "piggy bank," which uses cards showing one, two, three, or four pennies. All the cards are dealt, and each player keeps his or her cards in a stack facedown, without looking at them.

The object of the game is to make five cents with two cards. The first player turns over the top card of his or her stack and discards this card in the middle of the table, faceup. If this card is a 4, and the second player turns over a 1, the latter can take the two cards and keep them in his or her "bank." If, however, the second player turns over a 3, this 3 has to be discarded in the middle of the table, faceup, and the turn passes to the third player. Play continues with each player in turn trying to make five cents, and the game ends when all cards are used up.

Children who play "piggy bank" every day get to the point of turning over a 4 and looking for a 1, or turning over a 2 and looking for a 3, and so on. This is a missing-addend game, but its advantage is that it does not require the reading of "4 + __ = 5."

A similar game is "tens with nine cards." In this game, thirty-six cards with numerals from 1 to 9 are used, with the first nine cards of the deck arranged as in **figure 2.** The object of the game is to find all the number pairs that equal 10, such as 9 and 1, 3 and 7, and two 5s. After taking all possible pairs, the first player fills up the empty spaces with cards from the deck, and the turn passes to the next player.

"Tens with nine cards" is also a missing-

FIGURE 2

An arrangement of the cards in the game "tens with nine cards"

9	3	2
5	4	1
4	5	7

addend game, and children in kindergarten and first grade get to the point of looking for such combinations as 9 + 1, 8 + 2, 7 + 3, and so on. This game can be made harder by arranging the nine cards facedown and changing the game to "tens concentration."

Another way of making "tens with nine cards" harder is to use a "go fish" format. In "go fish" (or "go ten"), a player has to know what to ask for if he or she has a 4, for example. Another advantage of "go ten" is that if David asks Megan for an 8, it is possible for everybody else to infer that David must have a 2. However, first graders usually do not make such inferences.

Games such as "tens with nine cards" and "go ten" have the additional advantage of strengthening the foundation for the knowledge of place value. Children who know all the combinations that make 10 easily deal with 9 + 6 by changing it to 10 + 5. When they see 17 + 6, likewise, they can immediately think of "17 + 3 + 3" or "10 + (7 + 3) + 3."

It is, of course, possible to play "piggy bank" with numerals to 9, and the object of the game would be to make 10 with two cards. The children can also vary the total from day to day and decide, for example, that today's total for "piggy bank" (or "go fish") will be eight.

Another missing-addend game is "punta," which uses sixty cards—ten each of numerals 1–6. All the cards are dealt, and the players take turns rolling two six-sided or ten-sided dice. If a 5 and a 4 are rolled, for instance, all the players try to use their cards to make a total of 9 in as many ways as possible, such as 6 + 3, 4 + 4 + 1, 3 + 3 + 3, 4 + 3 + 2, and so on. The first person to use all his or her cards is the winner. An advantage of this game is that it encourages children to think flexibly and to build a rich network of numerical relationships, some of which are illustrated in **figure 3.** "Punta" also offers the possibility of thinking about strategies. For example, some children quickly notice the advantage of using the big numbers first to have the small numbers, such as 1 and 2, available at the end of the game.

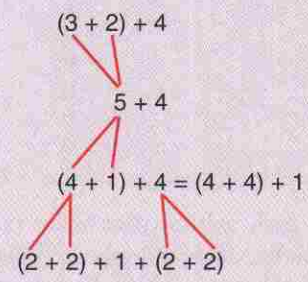
Children think very hard and from many viewpoints when they play games. They are also motivated to remember the combinations that make 5, 8, 10, and so on. Unlike standard worksheet activities, games keep children focused on numerical reasoning rather than on counting and writing.

What Is Hard about Missing-Addend Problems?

Such problems as 3 + __ = 5 are hard for many first graders not because they cannot do the numerical reasoning but because they cannot understand the problem. The eight children in our group test who had no idea what to write in the boxes could play the aforementioned games—not as well as the others in the same class according to their teachers, but they at least understood that they had to make a certain total. The difficulty of missing addends for these children, then, was in understanding the written form of the problem.

FIGURE 3

A network of numerical relationships that children can develop while playing "punta"



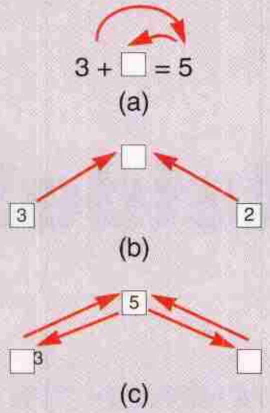
As shown in **figure 4a,** reading "3 + __ = 5" with understanding requires thinking about a part (3), the whole (5), and the missing part, simultaneously. The ability to think in two opposite directions simultaneously is what Piaget called *reversibility.* As Inhelder and Piaget (1964) demonstrated, reversibility usually develops around the age of seven or eight. Children who have developed reversibility can interpret "3 + __ = 5" as a part-whole relationship. Those who have not often write "8" as the answer, by adding the 3 and the 5.

For adults, "3 + __ = 5" is not any harder to understand than "3 + 2 = __ " because our thinking is reversible. For young children, however, "3 + 2 = __ " is much easier because it can be read by thinking in one direction, from the parts to the whole (see **fig. 4b**). **Figure 4c** is another way of showing that reading "3 + __ = 5" requires reversibility of thought.

Such games as "punta" encourage children to think flexibly. For example, children may think about 4 + 5 in the following ways: *(a)* 4 + 5 = 5 + 4 (switching the addends around); *(b)* 4 + 5 = 6 + 3 (switching the addends around *and* adding 1 to 5 and subtracting 1 from 4); *(c)* 6 + 3 = (3 + 3) + 3 (splitting 6 into 3 + 3); *(d)* 6 + 3 = (4 + 2) + 3 (splitting 6 into 4 + 2), and so on. This kind of flexible thinking later becomes reversible, and this reversibility in turn enables children to read missing-addend problems correctly.

For children whose thinking is not yet reversible, "piggy bank" is easier to deal with than the written form "3 + __ = 5" because, in the game, they have only one numeral to read at a time. If they turn over a 3, for example, all they have to read is the "3."

FIGURE 4 **The thinking involved in reading and understanding 3 + 2 = □ and 3 + □ = 5**



They can then figure out that they need a 2 and look for it. When they see "3 + __ = 5," however, they have to read two numerals with reversibility of thought if they are to understand the problem.

Conclusion

It will be recalled that eight children could not answer our questions in first grade. The four we could find in second grade were able to solve the missing-addend problems nine or ten months later. We believe that our hypothesis, that these children's thinking advanced to a level of reversibility without formal instruction, was confirmed.

When we read words, numerals, or mathematical symbols, we can get out of the marks on paper only the meaning that our thinking enables us to put into them. Children become able to read and understand missing-addend problems when their thinking becomes reversible.

A few games were described in this article, but we do not want to leave the impression that playing only these games will make children able to solve missing-addend problems. Children's numerical thinking is part of their thinking in general, and we do not recommend zeroing in only on missing addends. Children who think logically all the time can also think logically with reversibility when they are presented with missing-addend problems. For this reason we recommend a mathematics curriculum that is part of a more general curriculum emphasizing critical thinking and debate in daily living and discussions of various ways of solving word problems (see Kamii [1985, 1989, 1994]).

"The basics," such as knowing how to deal with 3 + __ = 5, are important. But we need to rethink how children acquire the "basics." It is often more productive to aim at a general goal, such as logical, numerical thinking that has depth, than to focus narrowly on specific "skills."

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