

Number Sense and Numeration, Grades 4 to 6

Volume 3 Multiplication

A Guide to Effective Instruction
in Mathematics,
Kindergarten to Grade 6

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Every effort has been made in this publication to identify mathematics resources and tools (e.g., manipulatives) in generic terms. In cases where a particular product is used by teachers in schools across Ontario, that product is identified by its trade name, in the interests of clarity. Reference to particular products in no way implies an endorsement of those products by the Ministry of Education.

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INTRODUCTION

Number Sense and Numeration, Grades 4 to 6 is a practical guide, in six volumes, that teachers will find useful in helping students to achieve the curriculum expectations outlined for Grades 4 to 6 in the Number Sense and Numeration strand of *The Ontario Curriculum, Grades 1–8: Mathematics, 2005*. This guide provides teachers with practical applications of the principles and theories that are elaborated in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006*.

The guide comprises the following volumes:

- Volume 1: The Big Ideas
- Volume 2: Addition and Subtraction
- Volume 3: Multiplication
- Volume 4: Division
- Volume 5: Fractions
- Volume 6: Decimal Numbers

The present volume – Volume 3: Multiplication – provides:

- a discussion of mathematical models and instructional strategies that support student understanding of multiplication;
- sample learning activities dealing with multiplication for Grades 4, 5, and 6.

A glossary that provides definitions of mathematical and pedagogical terms used throughout the six volumes of the guide is included in Volume 1: The Big Ideas. Each volume also contains a comprehensive list of references for the guide.

The content of all six volumes of the guide is supported by “eLearning modules” that are available at www.eworkshop.on.ca. The instructional activities in the eLearning modules that relate to particular topics covered in this guide are identified at the end of each of the learning activities (see pp. 43, 57, and 69).

Relating Mathematics Topics to the Big Ideas

The development of mathematical knowledge is a gradual process. A continuous, cohesive program throughout the grades is necessary to help students develop an understanding of the “big ideas” of mathematics – that is, the interrelated concepts that form a framework for learning mathematics in a coherent way.

(The Ontario Curriculum, Grades 1–8: Mathematics, 2005, p. 4)

In planning mathematics instruction, teachers generally develop learning opportunities related to curriculum topics, such as fractions and division. It is also important that teachers design learning opportunities to help students understand the big ideas that underlie important mathematical concepts. The big ideas in Number Sense and Numeration for Grades 4 to 6 are:

- quantity
- representation
- operational sense
- proportional reasoning
- relationships

Each of the big ideas is discussed in detail in Volume 1 of this guide.

When instruction focuses on big ideas, students make connections within and between topics, and learn that mathematics is an integrated whole, rather than a compilation of unrelated topics. For example, in a learning activity about division, students can learn about the relationship between multiplication and division, thereby deepening their understanding of the big idea of operational sense.

The learning activities in this guide do not address all topics in the Number Sense and Numeration strand, nor do they deal with all concepts and skills outlined in the curriculum expectations for Grades 4 to 6. They do, however, provide models of learning activities that focus on important curriculum topics and that foster understanding of the big ideas in Number Sense and Numeration. Teachers can use these models in developing other learning activities.

The Mathematical Processes

The Ontario Curriculum, Grades 1–8: Mathematics, 2005 identifies seven mathematical processes through which students acquire and apply mathematical knowledge and skills. The mathematical processes that support effective learning in mathematics are as follows:

- problem solving
- connecting
- reasoning and proving
- representing
- reflecting
- communicating
- selecting tools and computational strategies

The learning activities described in this guide demonstrate how the mathematical processes help students develop mathematical understanding. Opportunities to solve problems, to reason mathematically, to reflect on new ideas, and so on, make mathematics meaningful for students. The learning activities also demonstrate that the mathematical processes are interconnected – for example, problem-solving tasks encourage students to represent mathematical ideas, to select appropriate tools and strategies, to communicate and reflect on strategies and solutions, and to make connections between mathematical concepts.

Problem Solving: Each of the learning activities is structured around a problem or inquiry. As students solve problems or conduct investigations, they make connections between new mathematical concepts and ideas that they already understand. The focus on problem solving and inquiry in the learning activities also provides opportunities for students to:

- find enjoyment in mathematics;
- develop confidence in learning and using mathematics;
- work collaboratively and talk about mathematics;
- communicate ideas and strategies;
- reason and use critical thinking skills;
- develop processes for solving problems;
- develop a repertoire of problem-solving strategies;
- connect mathematical knowledge and skills with situations outside the classroom.

Reasoning and Proving: The learning activities described in this guide provide opportunities for students to reason mathematically as they explore new concepts, develop ideas, make mathematical conjectures, and justify results. The learning activities include questions that teachers can use to encourage students to explain and justify their mathematical thinking, and to consider and evaluate the ideas proposed by others.

Reflecting: Throughout the learning activities, students are asked to think about, reflect on, and monitor their own thought processes. For example, questions posed by the teacher encourage students to think about the strategies they use to solve problems and to examine mathematical ideas that they are learning. In the Reflecting and Connecting part of each learning activity, students have an opportunity to discuss, reflect on, and evaluate their problem-solving strategies, solutions, and mathematical insights.

Selecting Tools and Computational Strategies: Mathematical tools, such as manipulatives, pictorial models, and computational strategies, allow students to represent and do mathematics. The learning activities in this guide provide opportunities for students to select tools (concrete, pictorial, and symbolic) that are personally meaningful, thereby allowing individual students to solve problems and represent and communicate mathematical ideas at their own level of understanding.

Connecting: The learning activities are designed to allow students of all ability levels to connect new mathematical ideas to what they already understand. The learning activity descriptions provide guidance to teachers on ways to help students make connections among concrete, pictorial, and symbolic mathematical representations. Advice on helping students connect procedural knowledge and conceptual understanding is also provided. The problem-solving experiences in many of the learning activities allow students to connect mathematics to real-life situations and meaningful contexts.

Representing: The learning activities provide opportunities for students to represent mathematical ideas using concrete materials, pictures, diagrams, numbers, words, and symbols. Representing ideas in a variety of ways helps students to model and interpret problem situations, understand mathematical concepts, clarify and communicate their thinking, and make connections between related mathematical ideas. Students' own concrete and pictorial representations of mathematical ideas provide teachers with valuable assessment information about student understanding that cannot be assessed effectively using paper-and-pencil tests.

Communicating: Communication of mathematical ideas is an essential process in learning mathematics. Throughout the learning activities, students have opportunities to express mathematical ideas and understandings orally, visually, and in writing. Often, students are asked to work in pairs or in small groups, thereby providing learning situations in which students talk about the mathematics that they are doing, share mathematical ideas, and ask clarifying questions of their classmates. These oral experiences help students to organize their thinking before they are asked to communicate their ideas in written form.

Addressing the Needs of Junior Learners

Every day, teachers make many decisions about instruction in their classrooms. To make informed decisions about teaching mathematics, teachers need to have an understanding of the big ideas in mathematics, the mathematical concepts and skills outlined in the curriculum document, effective instructional approaches, and the characteristics and needs of learners.

The following table outlines general characteristics of junior learners, and describes some of the implications of these characteristics for teaching mathematics to students in Grades 4, 5, and 6.

Characteristics of Junior Learners and Implications for Instruction

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Intellectual development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • prefer active learning experiences that allow them to interact with their peers; • are curious about the world around them; • are at a concrete operational stage of development, and are often not ready to think abstractly; • enjoy and understand the subtleties of humour. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • learning experiences that allow students to actively explore and construct mathematical ideas; • learning situations that involve the use of concrete materials; • opportunities for students to see that mathematics is practical and important in their daily lives; • enjoyable activities that stimulate curiosity and interest; • tasks that challenge students to reason and think deeply about mathematical ideas.
Physical development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • experience a growth spurt before puberty (usually at age 9–10 for girls, at age 10–11 for boys); • are concerned about body image; • are active and energetic; • display wide variations in physical development and maturity. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • opportunities for physical movement and hands-on learning; • a classroom that is safe and physically appealing.
Psychological development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • are less reliant on praise but still respond well to positive feedback; • accept greater responsibility for their actions and work; • are influenced by their peer groups. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • ongoing feedback on students' learning and progress; • an environment in which students can take risks without fear of ridicule; • opportunities for students to accept responsibility for their work; • a classroom climate that supports diversity and encourages all members to work cooperatively.
Social development	<p>Generally, students in the junior grades:</p> <ul style="list-style-type: none"> • are less egocentric, yet require individual attention; • can be volatile and changeable in regard to friendship, yet want to be part of a social group; • can be talkative; • are more tentative and unsure of themselves; • mature socially at different rates. 	<p>The mathematics program should provide:</p> <ul style="list-style-type: none"> • opportunities to work with others in a variety of groupings (pairs, small groups, large group); • opportunities to discuss mathematical ideas; • clear expectations of what is acceptable social behaviour; • learning activities that involve all students regardless of ability. <p style="text-align: right;"><i>(continued)</i></p>

Characteristics of Junior Learners and Implications for Instruction

Area of Development	Characteristics of Junior Learners	Implications for Teaching Mathematics
Moral and ethical development	Generally, students in the junior grades: <ul style="list-style-type: none">• develop a strong sense of justice and fairness;• experiment with challenging the norm and ask “why” questions;• begin to consider others’ points of view.	The mathematics program should provide: <ul style="list-style-type: none">• learning experiences that provide equitable opportunities for participation by all students;• an environment in which all ideas are valued;• opportunities for students to share their own ideas and evaluate the ideas of others.

(Adapted, with permission, from *Making Math Happen in the Junior Grades*. Elementary Teachers’ Federation of Ontario, 2004.)

LEARNING ABOUT MULTIPLICATION IN THE JUNIOR GRADES

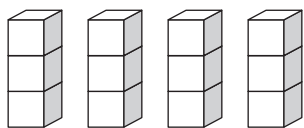
Introduction

The development of multiplication concepts represents a significant growth in students' mathematical thinking. With an understanding of multiplication, students recognize how groups of equal size can be combined to form a whole quantity. Developing a strong understanding of multiplication concepts in the junior grades builds a foundation for comprehending division concepts, proportional reasoning, and algebraic thinking.



PRIOR LEARNING

In the primary grades, students explore the meaning of multiplication by combining groups of equal size. Initially, students count objects one by one to determine the product in a multiplication situation. For example, students might use interlocking cubes to represent a problem involving four groups of three, and then count each cube to determine the total number of cubes.



With experience, students learn to use more sophisticated counting and reasoning strategies, such as using skip counting and using known addition facts (e.g., for 3 groups of 6: 6 plus 6 is 12, and 6 more is 18). Later, students develop strategies for learning basic multiplication facts, and use these facts to perform multiplication computations efficiently.

KNOWLEDGE AND SKILLS DEVELOPED IN THE JUNIOR GRADES

In the junior grades, instruction should focus on developing students' understanding of multiplication concepts and meaningful computational strategies, rather than on having students memorize the steps in algorithms. Learning experiences need to contribute to students'

understanding of part-whole relationships – that is, groups of equal size (the parts) can be combined to create a new quantity (the whole).

Instruction that is based on meaningful and relevant contexts helps students to achieve the curriculum expectations related to multiplication, listed in the following table.

Curriculum Expectations Related to Multiplication, Grades 4, 5, and 6		
By the end of Grade 4, students will:	By the end of Grade 5, students will:	By the end of Grade 6, students will:
<p>Overall Expectations</p> <ul style="list-style-type: none"> • solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies; • demonstrate an understanding of proportional reasoning by investigating whole-number unit rates. <p>Specific Expectations</p> <ul style="list-style-type: none"> • multiply to 9×9 and divide to $81 \div 9$, using a variety of mental strategies; • solve problems involving the multiplication of one-digit whole numbers, using a variety of mental strategies; • multiply whole numbers by 10, 100, and 1000, and divide whole numbers by 10 and 100, using mental strategies; • multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools, student-generated algorithms, and standard algorithms; • use estimation when solving problems involving the addition, subtraction, and multiplication of whole numbers, to help judge the reasonableness of a solution; • describe relationships that involve simple whole-number multiplication; • demonstrate an understanding of simple multiplicative relationships involving unit rates, through investigation using concrete materials and drawings. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> • solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies; • demonstrate an understanding of proportional reasoning by investigating whole-number rates. <p>Specific Expectations</p> <ul style="list-style-type: none"> • solve problems involving the addition, subtraction, and multiplication of whole numbers, using a variety of mental strategies; • multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms; • multiply decimal numbers by 10, 100, 1000, and 10 000, and divide decimal numbers by 10 and 100, using mental strategies; • use estimation when solving problems involving the addition, subtraction, multiplication, and division of whole numbers, to help judge the reasonableness of a solution; • describe multiplicative relationships between quantities by using simple fractions and decimals; • demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation using concrete materials and drawings. 	<p>Overall Expectations</p> <ul style="list-style-type: none"> • solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies; • demonstrate an understanding of relationships involving per cent, ratio, and unit rate. <p>Specific Expectations</p> <ul style="list-style-type: none"> • use a variety of mental strategies to solve addition, subtraction, multiplication, and division problems involving whole numbers; • solve problems involving the multiplication and division of whole numbers (four-digit by two-digit), using a variety of tools and strategies; • multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators; • multiply whole numbers by 0.1, 0.01, and 0.001 using mental strategies; • multiply and divide decimal numbers by 10, 100, 1000, and 10 000 using mental strategies.

(The Ontario Curriculum, Grades 1–8, Mathematics, 2005)

The following sections explain content knowledge related to multiplication concepts in the junior grades, and provide instructional strategies that help students develop an understanding of multiplication. Teachers can facilitate this understanding by helping students to:

- interpret multiplication situations;
- use models to represent multiplication;
- learn basic multiplication facts;
- develop skills in multiplying by multiples of 10;
- develop a variety of computational strategies;
- develop strategies for multiplying decimal numbers;
- develop effective estimation strategies for multiplication;
- relate multiplication and division.

Interpreting Multiplication Situations

Solving a variety of multiplication problems helps students to understand how the operation can be applied in different situations. Types of multiplication problems include equal-group problems and multiplicative-comparison problems.

Equal-group problems involve combining sets of equal size.

Examples:

- In a classroom, each work basket contains 5 markers. If there are 6 work baskets, how many markers are there?
- How many eggs are there in 3 dozen?
- Kendra bought 4 packs of stickers. Each pack cost \$1.19. How much did she pay?

Multiplicative-comparison problems involve a comparison between two quantities in which one is described as a multiple of the other. In multiplicative-comparison problems, students must understand expressions such as “3 times as many”. This type of problem helps students to develop their ability to reason proportionally.

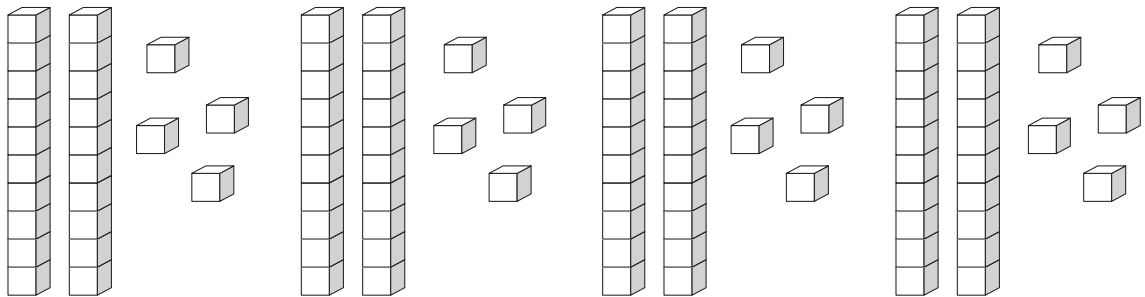
Examples:

- Luke’s dad is four times older than Luke is. If Luke is 9 years old, how old is his dad?
- Last Tuesday there was 15 cm of snow on the ground. The amount of snow has tripled since then. About how much snow is on the ground now?
- Felipe’s older sister is trying to save money. This month she saved 5 times as much money as she did last month. Last month she saved \$5.70. How much did she save this month?

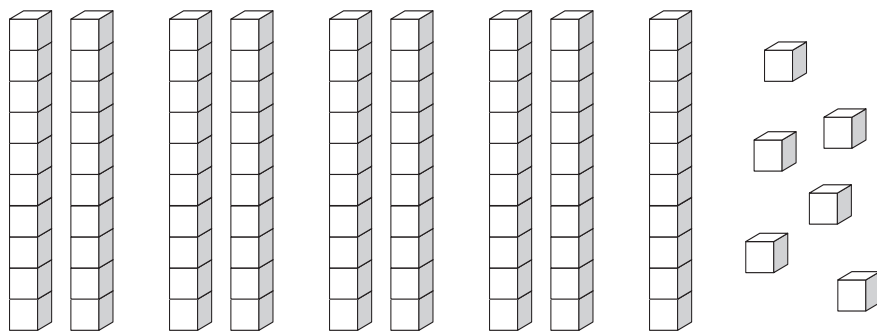
Students require experiences in interpreting both types of problems and in applying appropriate problem-solving strategies. It is not necessary, though, that students be able to identify or define these problem types.

Using Models to Represent Multiplication

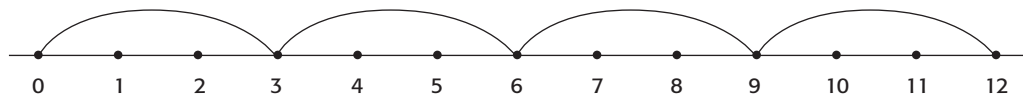
Models are concrete and pictorial representations of mathematical ideas. It is important that students have opportunities to represent multiplication using materials such as counters, interlocking cubes, and base ten blocks. For example, students might use base ten blocks to represent a problem involving 4×24 .



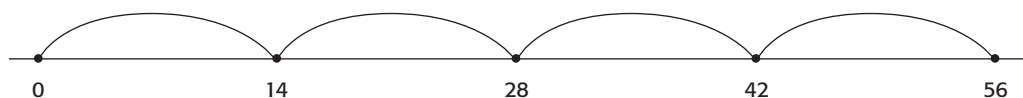
By regrouping the materials into tens and ones (and trading 10 ones cubes for a tens rod), students determine the total number of items.



Students can also model multiplication situations on number lines. Jumps of equal length on a number line reflect skip counting – a strategy that students use in early stages of multiplying. For example, a number line might be used to compute 4×3 .



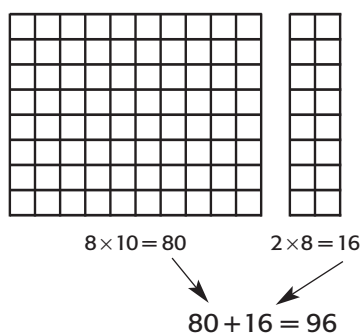
Later, students can use open number lines (number lines on which only significant numbers are indicated) to show multiplication with larger numbers. The following number line shows 4×14 .



An array (an arrangement of objects in rows and columns) provides a useful model for multiplication. In an array, the number of items in each row represents one of the factors in the multiplication expression, while the number of columns represents the other factor. Consider the following problem.

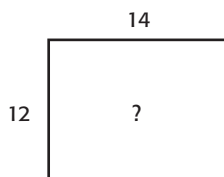
“Amy’s uncle has a large stamp collection. Her uncle displayed all his stamps from Australia on a large sheet of paper. Amy noticed that there were 8 rows of stamps with 12 stamps in each row. How many Australian stamps are there?”

To solve this problem, students might arrange square tiles in an array, and use various strategies to determine the number of tiles. For example, they might count the tiles individually, skip count groups of tiles, add 8 twelve times, or add 12 eight times. Students might also observe that the array can be split into two parts: an 8×10 part and an 8×2 part. In doing so, they decompose 8×12 into two multiplication expressions that are easier to solve, and then add the partial products to determine the product for 8×12 .



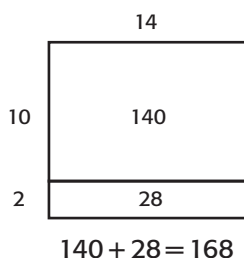
After students have had experiences with representing multiplication using arrays (e.g., making concrete arrays using tiles; drawing pictorial arrays on graph paper), teachers can introduce open arrays as a model for multiplication. In an open array, the squares or individual objects are not indicated within the interior of the array rectangle; however, the factors of the multiplication expression are recorded on the length and width of the rectangle. An open array does not have to be drawn to scale. Consider this problem.

“Eli helped his aunt make 12 bracelets for a craft sale. They strung 14 beads together to make each bracelet. How many beads did they use?”



The open array may not represent how students visualize the problem (i.e., the groupings of beads), nor does it provide an apparent solution to 12×14 . The open array does, however, provide a tool with which students can reason their way to a solution. Students might realize that

10 bracelets of 14 beads would include 140 beads, and that the other two bracelets would include 28 beads ($2 \times 14 = 28$). By adding $140 + 28$, students are able to determine the product of 12×14 .



The splitting of an array into parts (e.g., dividing a 12×14 array into two parts: 10×14 and 2×14) is an application of the distributive property. The property allows a factor in a multiplication expression to be decomposed into two or more numbers, and those numbers can be multiplied by the other factor in the multiplication expression.

Initially, mathematical models, such as open arrays, are used by students to represent problem situations and their mathematical thinking. With experience, students can also learn to use models as powerful tools with which to think (Fosnot & Dolk, 2001). Appendix 3–1: Using Mathematical Models to Represent Multiplication provides guidance to teachers on how they can help students use models as representations of mathematical situations, as representations of mathematical thinking, and as tools for learning.

Learning Basic Multiplication Facts

A knowledge of basic multiplication facts supports students in understanding multiplication concepts, and in carrying out more complex computations with multidigit multiplication. Students who do not have quick recall of facts often get bogged down and become frustrated when solving a problem. It is important to note that recall of multiplication facts does not necessarily indicate an understanding of multiplication concepts. For example, a student may have memorized the fact $5 \times 6 = 30$ but cannot create their own multiplication problem requiring the multiplication of five times six.

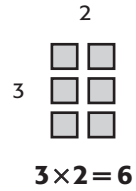
The use of models and thinking strategies helps students to develop knowledge of basic facts in a meaningful way. Chapter 10 in *A Guide to Effective Instruction in Mathematics, Kindergarten to Grade 6, 2006* (Volume 5) provides practical ideas on ways to help students learn basic multiplication facts.

Developing Skills in Multiplying by Multiples of 10

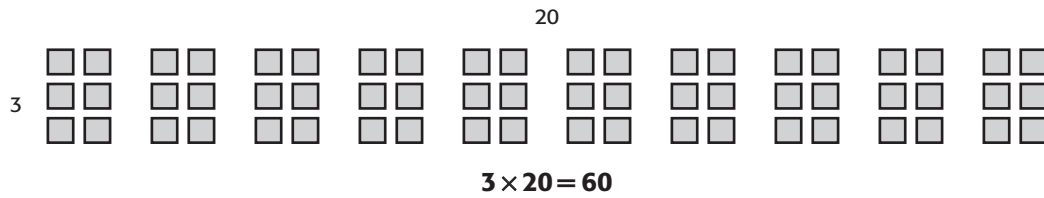
Because many strategies for multidigit multiplication depend on decomposing numbers to hundreds, tens, and ones, it is important that students develop skill in multiplying numbers by multiples of 10. For example, students in the junior grades should recognize patterns such as $7 \times 8 = 56$, $7 \times 80 = 560$, $7 \times 800 = 5600$, and $7 \times 8000 = 56\,000$.

Students can use models to develop an understanding of why patterns emerge when multiplying by multiples of 10. Consider the relationship between 3×2 and 3×20 .

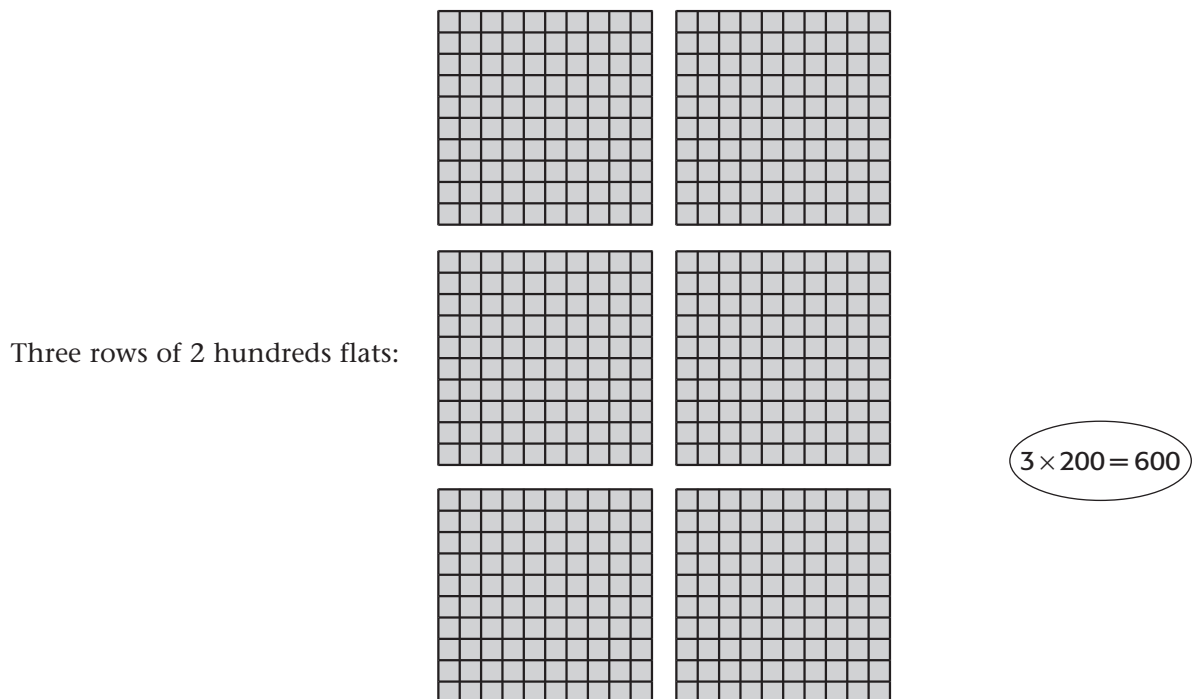
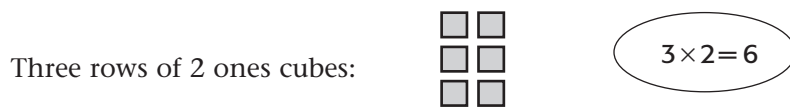
An array can be used to show 3×2 :



By arranging ten 3×2 arrays in a row, 3×20 can be modelled using an array. The array shows that 3×20 is also 10 groups of 6, or 60.



Students can also use base ten materials to model the effects of multiplying by multiples of 10. The following example illustrates 3×2 , 3×20 , and 3×200 .



Understanding the effects of multiplying by multiples of 10 also helps students to solve problems such as 30×40 , where knowing that $3 \times 4 = 12$ and $3 \times 40 = 120$ helps them to know that $30 \times 40 = 1200$.

Developing a Variety of Computational Strategies

Traditional approaches to teaching computation may generate beliefs about mathematics that impede further learning. These beliefs include fallacies such as the notion that only “smart” students can do math; that you must be able to do math quickly to do it well; and that math doesn’t need to be understood – you just need to follow the steps to get the answer. Research indicates that these beliefs begin to form during the elementary school years if the focus is on the mastery of standard algorithms, rather than on the development of conceptual understanding (Baroody & Ginsburg, 1986; Cobb, 1985; Hiebert, 1984).

There are numerous strategies for multiplication, which vary in efficiency and complexity. Perhaps the most complex (but not always most efficient) is the standard algorithm, which is quite difficult for students to use and understand if they have not had opportunities to explore their own strategies. For example, a common error is to misalign numbers when using the algorithm, as shown below:

$$\begin{array}{r} 125 \\ \times 12 \\ \hline 250 \\ \underline{125} \\ 375 \end{array}$$

A student who understands multiplication conceptually will recognize that this answer is not plausible. 125×10 is 1250, so multiplying 125×12 should result in a much greater product than 375.

While the following section provides a possible continuum for teaching multiplication strategies, it is important to note that there is no “culminating” strategy – teaching the standard algorithm for multiplication should not be the ultimate teaching goal for students in the junior grades. Students need to learn the importance of looking at the numbers in the problem, and then making decisions about which strategies are appropriate and efficient in given situations.

EARLY STRATEGIES FOR MULTIPLICATION PROBLEMS

Students are able to solve multiplication problems long before they are taught procedures for doing so. When students are presented with problems in meaningful contexts, they rely on strategies that they already understand to work towards a solution. For example, to solve a problem that involves 8 groups of 5, students might arrange counters into groups of 5, and then skip count by 5’s to determine the total number of counters.

Students might also use strategies that involve addition.

“A baker makes 48 cookies at a time. If the baker makes 6 batches of cookies each day, how many cookies does she make?”

Two possible approaches, both using addition, are shown below:

$$\begin{array}{r}
 48 \\
 + 48 \\
 \hline
 96 \\
 + 48 \\
 \hline
 144 \\
 + 48 \\
 \hline
 192 \\
 + 48 \\
 \hline
 240 \\
 + 48 \\
 \hline
 288
 \end{array}$$

$$\begin{array}{r}
 48 \quad 48 \quad 48 \\
 + 48 \quad + 48 \quad + 48 \\
 \hline
 96 \quad 96 \quad 96 \\
 \hline
 192 \\
 \hline
 288
 \end{array}$$

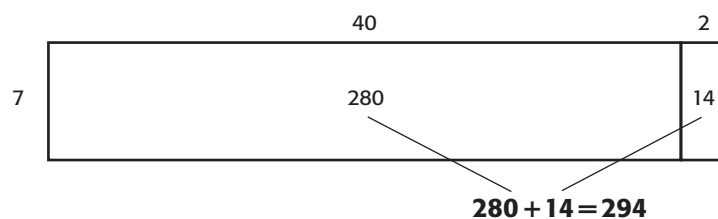
As students develop concepts about multiplication, and as their knowledge of basic facts increases, they begin to use multiplicative rather than additive strategies to solve multiplication problems.

PARTIAL PRODUCT STRATEGIES

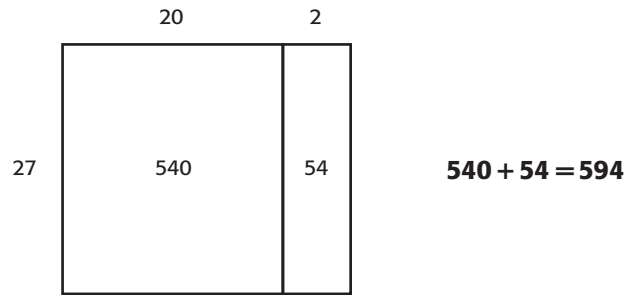
With partial product strategies, one or both factors in a multiplication expression are decomposed into two or more numbers, and these numbers are multiplied by the other factor. The partial products are added to determine the product of the original multiplication expression. Partial product strategies are applications of the distributive property of multiplication; for example, $5 \times 19 = (5 \times 10) + (5 \times 9)$. The following are examples of partial product strategies.

By Tens and Ones	By Place Value
<p>To compute 38×9, decompose 38 into $10 + 10 + 10 + 8$, then multiply each number by 9, and then add the partial products.</p> $ \begin{array}{r} 10 \times 9 = 90 \\ 10 \times 9 = 90 \\ 10 \times 9 = 90 \\ 8 \times 9 = 72 \\ \hline 270 \\ + 72 \\ \hline 342 \end{array} $	<p>To compute 278×8, decompose 278 into $200 + 70 + 8$, then multiply each number by 8, and then add the partial products.</p> $ \begin{array}{r} 200 \times 8 = 1600 \\ 70 \times 8 = 560 \\ 8 \times 8 = \underline{64} \\ \hline 2224 \end{array} $

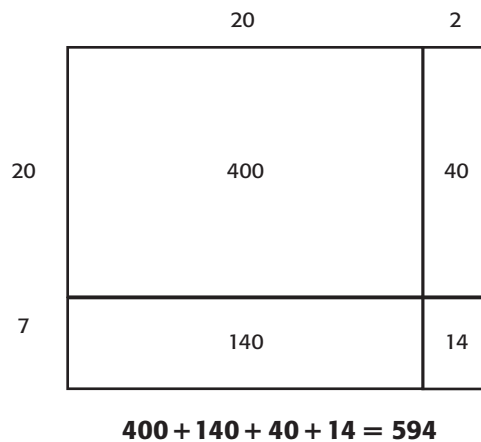
An open array provides a model for demonstrating partial product strategies, and gives students a visual reference for keeping track of the numbers while performing the computations. The following example shows how 7×42 might be represented using an open array.



An open array can also be used to multiply a two-digit number by a two-digit number. For example, to compute 27×22 , students might only decompose the 22.



Other students might decompose both 27 and 22, and use an array to show all four partial products.



Although the strategies described above rely on an understanding of the distributive property, it is not essential that students know the property by a rigid definition. What is important for them to know is that numbers in a multiplication expression can be decomposed to “friendlier” numbers, and that partial products can be added to determine the product of the expression.

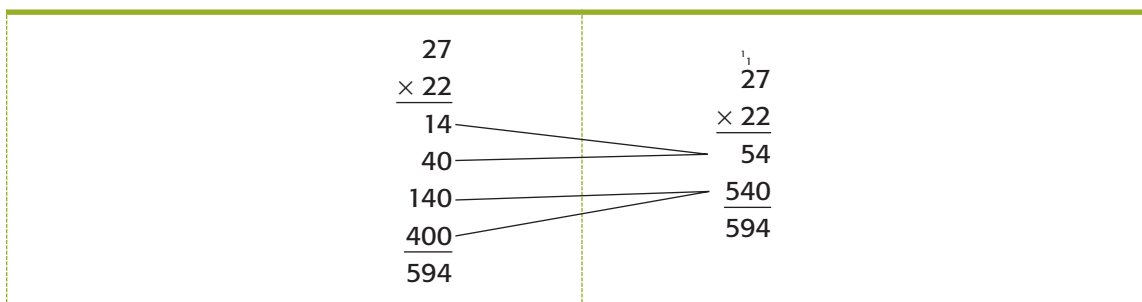
PARTIAL PRODUCT ALGORITHMS

Students benefit from working with a partial product algorithm before they are introduced to the standard multiplication algorithm. Working with open arrays, as explained above, helps students to understand how numbers can be decomposed in multiplication. The partial product algorithm provides an organizer in which students record partial products, and then add them to determine the final product. The algorithm helps students to think about place value and the position of numbers in their proper place-value columns.

$ \begin{array}{r} 27 \\ \times 22 \\ \hline (2 \times 7) \quad 14 \\ (2 \times 20) \quad 40 \\ (20 \times 7) \quad 140 \\ (20 \times 20) \quad \underline{200} \\ 594 \end{array} $	<p>Each partial product is recorded in the algorithm:</p> <ul style="list-style-type: none"> • ones \times ones • ones \times tens • tens \times ones • tens \times tens <p>The partial products are added to compute the final product.</p>
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STANDARD MULTIPLICATION ALGORITHM

When introducing the standard multiplication algorithm, it is helpful for students to connect it to the partial product algorithm. Students can match the numbers in the standard algorithm to the partial products.



OTHER MULTIPLICATION STRATEGIES

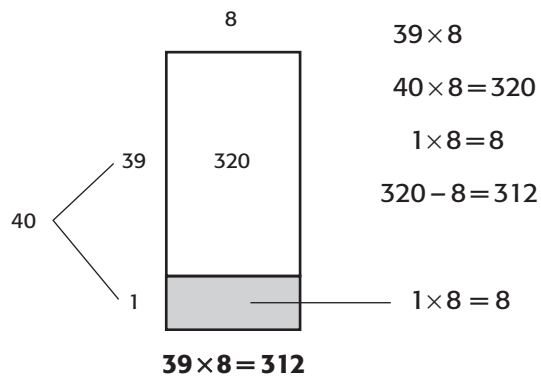
The ability to perform computations efficiently depends on an understanding of various strategies, and on the ability to select appropriate strategies in different situations. When selecting a computational strategy, it is important to examine the numbers in the problem first, in order to determine ways in which the numbers can be computed easily. Students need opportunities to explore various strategies and to discuss how different strategies can be used appropriately in different situations.

It is important that students develop an understanding of the strategies through carefully planned problems. An approach to the development of these strategies is through mini-lessons involving “strings” of questions. (See Appendix 2–1: Developing Computational Strategies Through Mini-Lessons, in Volume 2: Addition and Subtraction.)

The following are some multiplication strategies for students to explore.

Compensation: A compensation strategy involves multiplying more than is needed, and then removing the “extra” at the end. This strategy is particularly useful when a factor is close to a multiple of 10. To multiply 39×8 , for example, students might recognize that 39 is close to 40, multiply 40×8 to get 320, and then subtract the extra 8 (the difference between 39×8 and 40×8).

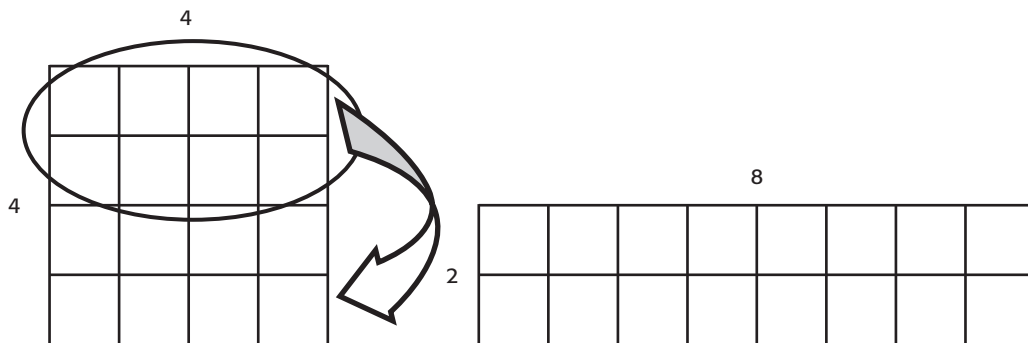
The compensation strategy can be modelled using an open array.



Regrouping: The associative property allows the factors in a multiplication expression to be regrouped without affecting the outcome of the product. For example, $(2 \times 3) \times 6 = 2 \times (3 \times 6)$.

Sometimes, when multiplying three or more factors, changing the order in which the factors are multiplied can simplify the computation. For example, the product of $2 \times 16 \times 5$ can be found by multiplying 2×5 first, and then multiplying 10×16 .

Halving and Doubling: Halving and doubling can be represented using an array model. For example, 4×4 can be modelled using square tiles arranged in an array. Without changing the number of tiles, the tiles can be rearranged to form a 2×8 array.



The length of the array has been doubled (4 becomes 8) and the width has been halved (4 becomes 2), but the product (16 tiles) is unchanged.

The halving-and-doubling strategy is practical for many types of multiplication problems that students in the junior grades will experience. The associative property can be used to illustrate how the strategy works.

$$\begin{aligned}
 26 \times 5 &= (13 \times 2) \times 5 \\
 &= 13 \times (2 \times 5) \\
 &= 13 \times 10 \\
 &= 1300
 \end{aligned}$$

In some cases, the halving-and-doubling process can be applied more than once to simplify a multiplication expression.

$$\begin{aligned}12 \times 15 &= 6 \times 30 \\ &= 3 \times 60 \\ &= 180\end{aligned}$$

When students are comfortable with halving and doubling, carefully planned activities will help them to generalize the strategy – that is, multiplying one number in the multiplication expression by a factor, and dividing the other number in the expression by the same factor, results in the same product as that for the original expression. Consequently, thirding and tripling, and fourthing and quadrupling are also possible computational strategies, as shown below.

Thirding and Tripling

$$\begin{array}{ccc}15 \times 12 & 18 \times 15 & 30 \times 16 \\ 5 \times 36 = 180 & 6 \times 45 = 270 & 10 \times 48 = 480\end{array}$$

Fourthing and Quadrupling

$$\begin{array}{ccc}12 \times 75 & 24 \times 250 & 16 \times 125 \\ 3 \times 300 = 900 & 6 \times 1000 = 6000 & 4 \times 500 = 2000\end{array}$$

Doubling: With the doubling strategy, a multiplication expression is simplified by reducing one of its factors by half. After computing the product for the simplified expression, the product is doubled. For example, to solve 6×15 , the student might think “ 3×15 is 45, so double that is 90.”

An advanced form of doubling involves factoring out the twos.

for 8×36 :

$$2 \times 36 = 72$$

$$2 \times 72 = 144$$

$$2 \times 144 = 288$$

Recognizing that $8 = 2 \times 2 \times 2$ helps students know when to stop doubling.

Developing Strategies for Multiplying Decimal Numbers

The ability to multiply by 10 and by powers of 10 helps students to multiply decimal numbers. When students know the effect that multiplying or dividing by 10 or 100 or 1000 has on a product, they can rely on whole-number strategies to multiply decimal numbers.

To solve a problem involving 7.8×8 , students might use the following strategy:

“Multiply 7.8 by 10, so that the multiplication involves only whole numbers. Next, multiply 78×8 to get 624. Then divide 624 by 10 (to “undo” the effect of multiplying 7.8 by 10 earlier). 7.8×8 is 62.4.”

Estimation plays an important role when multiplying two- and three-digit decimal numbers. For example, to calculate 38.8×9 , students should recognize that 38.8×9 is close to 40×9 , and estimate that the product will be close to 360.

Then, students perform the calculation using whole numbers, ignoring the decimal point.

$$\begin{array}{r} 388 \\ \times 9 \\ \hline 72 \\ 720 \\ \hline 2700 \\ 3492 \end{array}$$

After completing the algorithm, students refer back to their estimate to make decisions about the correct placement of the decimal point (e.g., based on the estimate, the only logical place for the decimal point is to the right of the 9, resulting in the answer 349.2).

Developing Estimation Strategies for Multiplication

Students should develop a range of effective estimation strategies, but they should also be aware of when one strategy is more appropriate than another. It is important for students to consider the context of the problem before selecting an estimation strategy. Students should also decide beforehand how accurate their estimate needs to be. Consider the following problem situation.

“The school secretary is placing an order for pencils for the next school year, and would like your help in figuring out how many pencils to order. She estimates that each student in the school will use about 6 pencils during the year. There will be approximately 225 students in the school. How many pencils should the school order?”

In this situation, students should estimate “on the high side” to ensure that enough, rather than too few, pencils are ordered. For example, they might multiply 250 and 6 to get an estimate of 1500.

Estimation is an important skill when solving problems involving multiplication, and there are many more strategies than simply rounding. The estimation strategies that students use for addition and subtraction may not apply to multiplication, and a firm conceptual understanding of multiplication is needed to estimate products efficiently.

The following table outlines different estimation strategies for multiplication. It is important to note that the word “rounding” is used loosely – it does not refer to any set of rules or procedures for rounding numbers (e.g., look to the number on the right; is it greater than 5 . . .).

Strategy	Example
Rounding one or both numbers to the nearest multiple of 10, 100, 1000, ...	15×32 is about $15 \times 30 = 450$ 28×49 is about $30 \times 50 = 1500$
Finding friendly numbers	5×27 is about $5 \times 25 = 125$
Rounding one factor up and the other factor down	43×18 is about $40 \times 20 = 800$
Using front-end estimation (Note that this strategy is less accurate with multiplication than with addition.)	125×46 is about $100 \times 40 = 4000$ (actual 5750)
Finding a range (by rounding one or both factors down, then up)	26×8 is about $20 \times 8 = 160$ 26×8 is about $30 \times 8 = 240$ The product is between 160 and 240.

It is important for students to know that with multiplication, rounding one factor can have a significantly different impact on the product than rounding the other. Consider the calculation $48 \times 8 = 384$. If students round 48 to 50, the estimation would be 400 (a difference of two 8's), which is very close to the actual product. If students round 8 to 10, the estimation would be 480 (a difference of two 48's), which is considerably farther from the actual product. Comparing the effects of rounding both factors will help develop students' understanding of quantity and operations.

Relating Multiplication and Division

Multiplication and division are inverse operations: multiplication involves combining groups of equal size to create a whole, whereas division involves separating the whole into equal groups. In problem-solving situations, students can be asked to determine the total number of items in the whole (multiplication), the number of items in each group (partitive division), or the number of groups (quotative division).

Students should experience problems such as the following, which allow them to see the connections between multiplication and division.

“Samuel needs to equally distribute 168 cans of soup to 8 shelters in the city. How many cans will each shelter get?”

“The cans come in cases of 8. How many cases will Samuel need in order to have 168 cans of soup?”

Although both problems seem to be division problems, students might solve the second one using multiplication – by recognizing that 20 cases would provide 160 cans ($20 \times 8 = 160$) and an additional case would be another 8 cans ($1 \times 8 = 8$), and therefore determining that 21 cases would provide 168 cans. With this strategy, students, in essence, decompose 168 into $(20 \times 8) + (1 \times 8)$, and then add $20 + 1 = 21$.

Providing opportunities to solve related problems helps students develop an understanding of the part-whole relationships inherent in multiplication and division situations, and enables them to use multiplication and division interchangeably depending on the problem.

A Summary of General Instructional Strategies

Students in the junior grades benefit from the following instructional strategies:

- experiencing a variety of multiplication problems, including equal-group and multiplicative-comparison problems;
- using concrete and pictorial models to represent mathematical situations, to represent mathematical thinking, and to use as tools for new learning;
- solving multiplication problems that serve different instructional purposes (e.g., to introduce new concepts, to learn a particular strategy, to consolidate ideas);
- providing opportunities to develop and practise mental computation and estimation strategies;
- providing opportunities to connect division to multiplication through problem solving.

The Grades 4–6 Multiplication and Division module at www.eworkshop.on.ca provides additional information on developing multiplication concepts with students. The module also contains a variety of learning activities and teaching resources.





APPENDIX 3-1: USING MATHEMATICAL MODELS TO REPRESENT MULTIPLICATION

The Importance of Mathematical Models

Models are concrete and pictorial representations of mathematical ideas, and their use is critical in order for students to make sense of mathematics. At an early age, students use models such as counters to represent objects and tally marks to keep a running count.

Standard mathematical models, such as number lines and arrays, have been developed over time and are useful as “pictures” of generalized ideas. In the junior grades, it is important for teachers to develop students’ understanding of a variety of models so that models can be used as tools for learning.

The development in understanding a mathematical model follows a three-phase continuum:

- **Using a model to represent a mathematical situation:** Students use a model to represent a mathematical problem. The model provides a “picture” of the situation.
- **Using a model to represent student thinking:** After students have discussed a mathematical idea, the teacher presents a model that represents students’ thinking.
- **Using a model as a tool for new learning:** Students have a strong understanding of the model and are able to apply it in new learning situations.

An understanding of mathematical models takes time to develop. A teacher may be able to take his or her class through only the first or second phase of a particular model over the course of a school year. In other cases, students may quickly come to understand how the model can be used to represent mathematical situations, and a teacher may be able to take a model to the third phase with his or her class.

USING A MODEL TO REPRESENT A MATHEMATICAL SITUATION

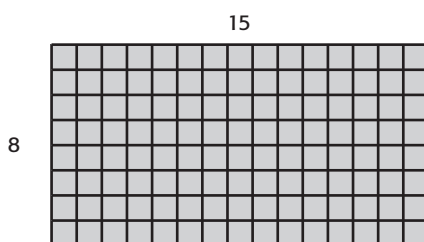
A well-crafted problem can lead students to use a mathematical model that the teacher would like to highlight. The following example illustrates how the use of an array as a model for multiplication might be introduced.

A teacher provides students with the following problem:

“I was helping my mother design her new rectangular patio. It will be made of square tiles. The long side of her patio will be 15 tiles long, and the short side will be 8 tiles long. How many square tiles should she buy?”

This problem was designed to encourage students to construct or draw arrays. The teacher purposefully included numbers that students could not multiply mentally.

After presenting the problem, the teacher encourages students to solve the problem in a way that makes sense to them. Some students use square tiles to recreate the patio, and then use repeated addition to determine the total number of tiles.



A student explains his strategy:

“First, I made the patio out of square tiles, and found out I had 8 rows of 15 tiles. So I added 15 eight times to get the total: $15 + 15 + 15 + 15 + 15 + 15 + 15 + 15 = 120$.”

Other students use similar strategies. For example, some students draw a diagram of the patio, and then add 8 fifteen times.

The teacher has not provided students with a particular model to solve the problem, but the context of the problem (creating a rectangular shape with square tiles) lends itself to using an array model. Although students used an array to represent the patio in the problem, they might not apply the array model in other multiplication problems. It is the teacher’s role to help students generalize the use of the array model to other multiplication situations.

USING A MODEL TO REPRESENT STUDENT THINKING

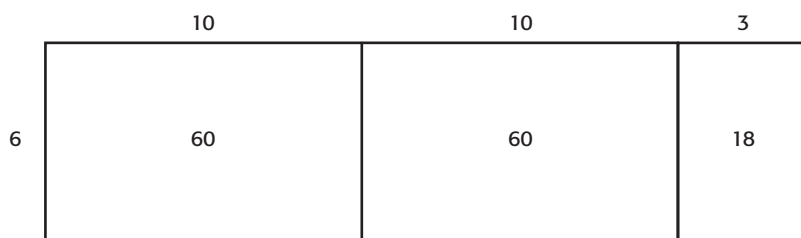
Teachers can guide students in recognizing how models can represent mathematical thinking. The following example provides an illustration.

A teacher is providing an opportunity for students to develop mental multiplication strategies. He asks his students to calculate a series of multiplication questions mentally: 6×10 , 6×20 , 6×3 , 6×23 .

A student explains her strategy for solving 6×23 :

“First I multiplied 6×10 to get 60. Then I multiplied 6×10 again because there is a 20 in 23. I added $60 + 60$ to get 120. So 6×20 is 120. But it’s 6×23 , not 6×20 , so I multiplied 6×3 to get 18. Then I added $18 + 120$ to get 138.”

The teacher takes this opportunity to represent the student’s thinking by drawing an open array on the board. (The open array does not have to be drawn to scale – the dividing lines simply represent the decomposition of a factor.)



$$60 + 60 + 18 = 138$$

The teacher uses the open array to discuss the strategy with the class. The diagram helps students to visualize how 23 is decomposed into 10, 10, and 3; then each “part” is multiplied by 6; and then the three partial products are added together.

By representing the computational strategy using an open array, the teacher shows how the array can be used to represent mathematical thinking. Given ongoing opportunities to use open arrays to represent computational strategies and solutions to problems, students will come to “own” the model and use the open array as a tool for learning.

USING A MODEL AS A TOOL FOR NEW LEARNING

To help students generalize the use of an open array as a model for multiplication, and to help them recognize its utility as a tool for learning, teachers need to provide problems that allow students to apply and extend the strategy of partial products.

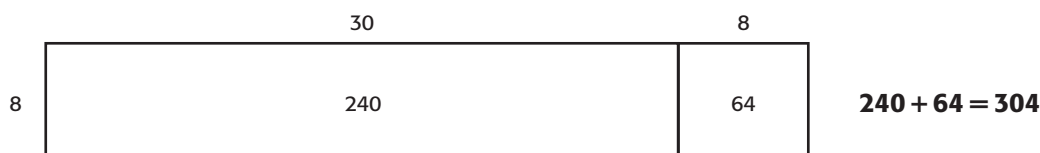
A teacher poses the following problem:

“The principal will be placing an order for school supplies, and he asked me to check the number of markers in the school’s supply cupboard. I counted 38 boxes, and I know that there are 8 markers in each box. I haven’t had time to figure out the total number of markers yet. Could you help me with this problem?”

Prior to this, the class investigated the use of open arrays and the distributive property in solving multiplication problems.

Several students use an open array to solve the problem – they decompose 38 into 30 and 8, then multiply both numbers by 8, and then add the partial products.

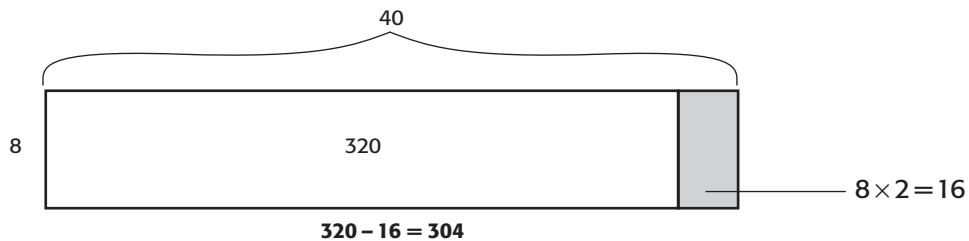
One student uses the array model in a different way. She explains her strategy:



“I know that 38 is close to 40, so I drew an array that was 40 long and 8 wide. I knew 8×40 is 320, but I didn’t need 40 eights – I only needed 38 eights, so I took 2 eights away at the end. $320 - 16 = 304$ ”

The student drew the following array to solve the problem:

This student used an array to apply the distributive property but extended the use of the model



to include a new compensation strategy – calculating more than is needed, and then subtracting the extra part.

In this case, the model has become a tool for learning. The student is not simply replicating a strategy used in previous problems, but instead uses it to solve a related problem in a new way.

When developing a model for multiplication, it is practical to assume that not all students will come to understand or use the model with the same degree of effectiveness. Teachers should continue to develop meaningful problems that allow students to use strategies that make sense to them. However, part of the teacher’s role is to use models to represent students’ ideas so that these models will eventually become thinking tools for students. The ability to generalize a model and use it as a learning tool takes time (possibly years) to develop.

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Learning Activities for Multiplication

Introduction

The following learning activities for Grades 4, 5, and 6 provide teachers with instructional ideas that help students achieve some of the curriculum expectations related to multiplication. The learning activities also support students in developing their understanding of the big ideas outlined in Volume 1: The Big Ideas.

The learning activities do not address all concepts and skills outlined in the curriculum document, nor do they address all the big ideas – one activity cannot fully address all concepts, skills, and big ideas. The learning activities demonstrate how teachers can introduce or extend mathematical concepts; however, students need multiple experiences with these concepts to develop a strong understanding.

Each learning activity is organized as follows:

OVERVIEW: A brief summary of the learning activity is provided.

BIG IDEAS: The big ideas that are addressed in the learning activity are identified. The ways in which the learning activity addresses these big ideas are explained.

CURRICULUM EXPECTATIONS: The curriculum expectations are indicated for each learning activity.

ABOUT THE LEARNING ACTIVITY: This section provides guidance to teachers about the approximate time required for the main part of the learning activity, as well as the materials, math language, instructional groupings, and instructional sequencing for the learning activity.

ABOUT THE MATH: Background information is provided about the mathematical concepts and skills addressed in the learning activity.

GETTING STARTED: This section provides the context for the learning activity, activates prior knowledge, and introduces the problem or activity.

WORKING ON IT: In this part, students work on the mathematical activity, often in small groups or with a partner. The teacher interacts with students by providing prompts and asking questions.

REFLECTING AND CONNECTING: This section usually includes a whole-class debriefing time that allows students to share strategies and the teacher to emphasize mathematical concepts.

ADAPTATIONS/EXTENSIONS: These are suggestions for ways to meet the needs of all learners in the classroom.

ASSESSMENT: This section provides guidance for teachers on assessing students' understanding of mathematical concepts.

HOME CONNECTION: This section is addressed to parents or guardians, and includes an activity for students to do at home that is connected to the mathematical focus of the main learning activity.

LEARNING CONNECTIONS: These are suggestions for follow-up activities that either extend the mathematical focus of the learning activity or build on other concepts related to the topic of instruction.

BLACKLINE MASTERS: These pages are referred to and used throughout the learning activities.

Grade 4 Learning Activity

Chairs, Chairs, and More Chairs!

OVERVIEW

In this learning activity, students solve a problem in which they determine the number of chairs arranged in a 7×24 array. The problem-solving experience provides an opportunity for students to explore a variety of multiplication strategies.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students solve a problem involving the multiplication of a two-digit number by a one-digit number using a variety of strategies (e.g., using repeated addition, using doubling, using the distributive property). The learning activity focuses on informal strategies that make sense to students, rather than on the teaching of multiplication algorithms.

Relationships: The learning activity allows students to recognize relationships between operations (e.g., the relationship between repeated addition and multiplication). Working with arrays also helps students to develop an understanding of how factors in a multiplication expression can be decomposed to facilitate computation. For example, by applying the distributive property, 7×24 can be decomposed into $(7 \times 20) + (7 \times 4)$.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- multiply two-digit whole numbers by one-digit whole numbers, using a variety of tools (e.g., base ten materials or drawings of them, arrays), student-generated algorithms, and standard algorithms.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- solve problems involving the addition, subtraction, multiplication, and division of single- and multidigit whole numbers, and involving the addition and subtraction of decimal numbers to tenths and money amounts, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

TIME:
approximately
60 minutes

MATERIALS

- sheets of paper (1 per group of 3 students)
- square tiles
- **Mult4.BLM1: Grid Paper** (1 per group of 3 students)
- sheets of chart paper or large sheets of newsprint (1 per group of 3 students)
- markers (a few per group of 3 students)
- glue (optional)
- scissors (optional)
- sheets of paper or math journals (1 per student)
- **Mult4.BLM2: Exploring Multiplication** (1 per student)

MATH LANGUAGE

- repeated addition
- multiplication
- array
- row
- column
- product
- partial products
- friendly number
- open array

INSTRUCTIONAL
GROUPING:
groups of 3

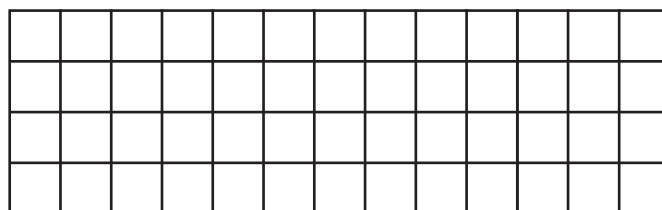
INSTRUCTIONAL SEQUENCING

This learning activity provides an introductory exploration of strategies for multiplying a two-digit number by a one-digit number. Before starting this activity, students should have an understanding of basic multiplication concepts (e.g., that multiplication involves combining groups of equal size), and some knowledge of basic facts (and of strategies to determine answers to unknown facts). Students should be given other opportunities to solve multiplication problems using their own strategies, before they investigate multiplication algorithms.

ABOUT THE MATH

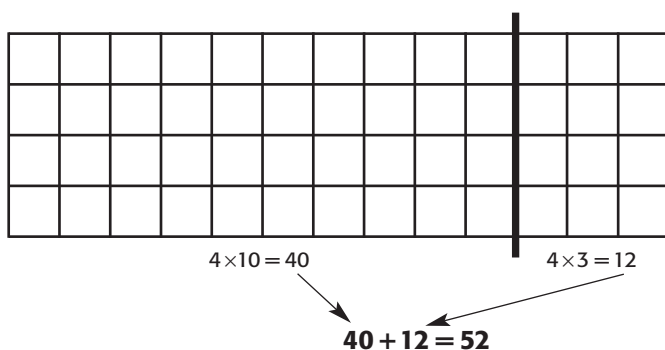
Students are capable of solving multiplication problems before they develop an understanding of algorithms. When students apply strategies that make sense to them, they develop a deeper understanding of the operation and of different multiplication strategies.

This learning activity allows students to explore multiplication by using an array. The organization of items in rows and columns allows students to observe arrays as models of multiplication.



4 rows of 13 → 4×13

Dividing arrays into parts is an effective way to show how the distributive property can be applied to facilitate multiplication.



Discussions about various multiplication strategies and the use of arrays to represent multiplication are important components of this learning activity. These conversations allow students to learn strategies from one another and to recognize the power of the array as a tool for representing multiplication.

GETTING STARTED

Describe the following scenario to the class:

“The custodian at our school needs to set up chairs in the gym for a parents’ meeting. He plans to arrange the chairs in 7 rows with 24 chairs in each row. He is wondering, though, whether there will be enough chairs for 150 parents. How many chairs will there be altogether? Will there be enough chairs?”

On the board, record important information about the problem:

- 7 rows
- 24 chairs in each row
- How many chairs altogether?
- Are there enough chairs for 150 parents?

Divide the class into groups of three. Ask students to work together to solve the problem in a way that makes sense to everyone in their group. Suggest that students use materials such as square tiles and grid paper. Provide each group with a sheet of paper on which students can record their work.

WORKING ON IT

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

- “What strategy are you using to solve the problem?”
- “Why are you using this strategy?”
- “Did you change or modify your strategy? Why?”
- “How are you representing the rows of chairs? Is this an effective way to represent the chairs?”

Students might use manipulatives (e.g., square tiles), draw on grid paper, or make a diagram to represent the arrangement of chairs. Concrete arrays and pictorial arrays help students to think of and apply strategies for determining the total number of chairs.

STRATEGIES STUDENTS MIGHT USE

COUNTING

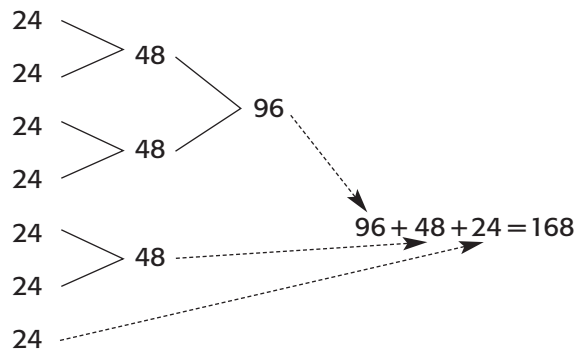
Although inefficient, counting the chairs is a strategy some students might use if they are not ready to consider the array as a representation of multiplication.

USING REPEATED ADDITION

The creation of a 7×24 array might prompt some students to use repeated addition – adding 7 twenty-four times, or adding 24 seven times.

DOUBLING

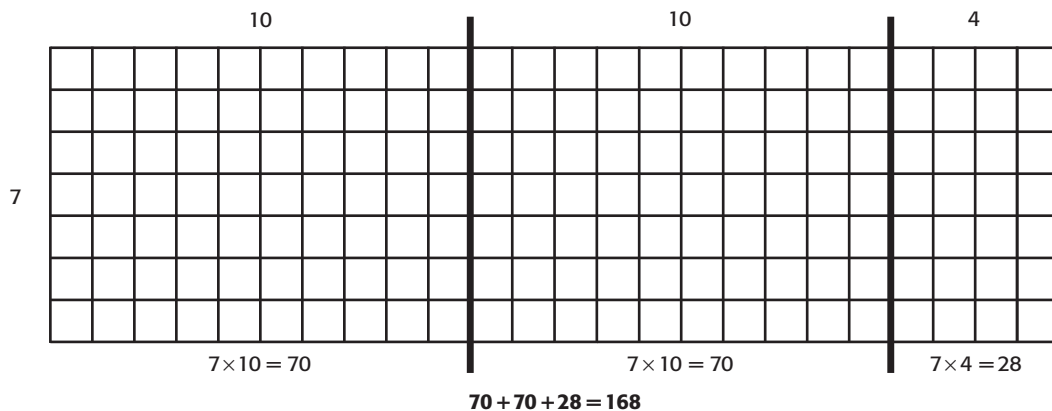
Students might use a doubling strategy similar to the following:



DECOMPOSING THE ARRAY (USING THE DISTRIBUTIVE PROPERTY)

Some students might decompose 7×24 into smaller parts, then use known multiplication facts to determine the products of the smaller parts, and then add the partial products to determine the total number of chairs. Students might divide the array into two or more parts without considering whether the resulting numbers can be easily calculated.

Other students might think about ways to divide the array to work with “friendly” numbers.



USING MENTAL COMPUTATION (APPLYING THE DISTRIBUTIVE PROPERTY)

Students might think of these steps:

- 7 rows of 20 chairs is 140 chairs.
- To account for the extra 4 chairs in each row, multiply 7×4 .
- $140 \text{ chairs} + 28 \text{ chairs} = 168 \text{ chairs}$

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheets of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem. If students used grid paper, they could cut out their arrays and glue them to the sheet of paper.

Make a note of groups who might share their strategies and solutions during Reflecting and Connecting. Include groups who used various methods that range in their degree of efficiency (e.g., counting; using repeated addition; using doubling; using the distributive property without considering whether the resulting numbers can be easily calculated; using the distributive property to find friendly numbers).

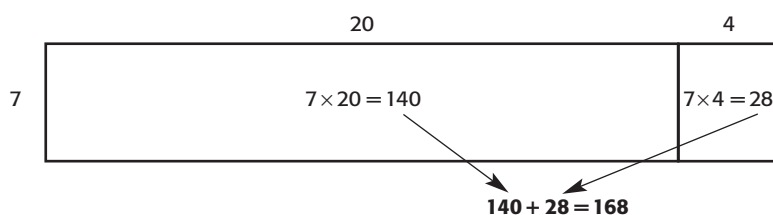
REFLECTING AND CONNECTING

Reconvene the class. Ask a few groups to share their problem-solving strategies and solution, and post their work. Try to order the presentations so that students observe inefficient strategies (e.g., counting, using repeated addition) first, followed by increasingly efficient methods.

As students explain their work, ask questions that help them to describe their strategies:

- “What strategy did you use to determine the total number of chairs?”
- “Why did you use this strategy?”
- “How does your strategy work?”
- “Was your strategy easy or difficult to use? Why?”
- “Would you use this strategy if you solved a problem like this again? Why or why not?”
- “How would you change your strategy the next time?”
- “How do you know that your solution is correct?”

If students describe a mental computation strategy that is based on the distributive property, you might model their thinking by drawing an open array (i.e., an array in which the interior squares are not indicated).



Following the presentations, ask students to observe the work that has been posted, and to consider the efficiency of the various strategies. Ask:

- “Which strategy, in your opinion, is an efficient strategy?”
- “Why is the strategy effective in solving this kind of problem?”
- “How would you explain this strategy to someone who has never used it?”

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students’ work to emphasize ideas about the distributive property:

- Arrays can be decomposed into two or more parts.
- The product of each part can be calculated and the partial products added together to determine the product of the entire array.
- An array can be decomposed into parts that provide friendly numbers, which are easy to calculate with.

Note: It is not necessary for students to define the “distributive property”, but they should learn how it can be applied to facilitate multiplication.

Provide an opportunity for students to solve a related problem. Explain that bottles of water will be set on a table for the parent meeting, and that the bottles will be arranged in 6 rows with 31 bottles in each row. Ask students to give the multiplication expression related to the problem. Record “ 6×31 ” on the board.

Have students work in groups of three. (You can use the same groups as before, or form different groups.) Encourage students, in their groups, to consider the various strategies that have been discussed, and to apply a method that will allow them to solve the problem efficiently. After groups have solved the problem, ask students, independently, to record a solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to use strategies that make sense to them. Recognize that some students may need to rely on simple strategies, such as counting or using repeated addition, and may not be ready to apply more sophisticated strategies. Ensure that students use concrete materials or a drawing to represent the multiplication situation and to connect it to an array.

Guide students in using more efficient strategies when you observe that they are ready to do so. For example, students who use a counting strategy could be encouraged to use repeated addition. Provide opportunities for students who experience difficulties to work with students who can support them in understanding the arrangement in an array and how it can be divided into smaller parts.

Challenge students to solve the problem in different ways. For example, if students use an algorithm, ask them to explain how the algorithm works and the meaning of the numbers in the algorithm within the context of the problem.

ASSESSMENT

Observe students as they solve the problem, and assess how well they:

- represent and explain the problem situation (e.g., using an array made with square tiles or grid paper, using a drawing);
- apply an appropriate strategy for solving the problem;
- explain their strategy and solution;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient ways to solve the problem;
- explain ideas about the distributive property (e.g., that 7×24 can be decomposed into $(7 \times 20) + (7 \times 4)$, and that the partial products, 140 and 28, can be added to calculate the final product).

Collect the math journals or sheets of paper on which students recorded their strategies and solutions for the water bottle problem. Observe students' work to determine how well they apply an efficient strategy for solving the multiplication problem.

HOME CONNECTION

Send home **Mult4.BLM2: Exploring Multiplication**. In the letter, parents are asked to review the multiplication strategies that are modelled for them. Next they are invited to assist their child as he or she explains how to calculate the product of 3×17 .

LEARNING CONNECTION 1

How Many Fruits?

MATERIALS

- overhead transparency of **Mult4.BLM3: How Many Fruits?**
- overhead projector
- sheets of paper (1 per student)

Display an overhead transparency of **Mult4.BLM3: How Many Fruits?** Discuss how fruits are often arranged in arrays in grocery stores. Ask students to describe the different arrays of fruits in the picture.

Next, challenge students to figure out the total number of pieces of fruit in the picture. Have them record their strategies and solution on a sheet of paper.

Observe students as they solve the problem, and make note of the different methods that they use. For example, students might:

- skip count;
- use multiplication to determine the number of pieces of fruit in each tray, and then add the six partial products;
- use multiplication to determine the number of pieces of each kind of fruit, and then add the three partial products;
- multiply the number of rows by the number of pieces of fruit in each row (6×9).

Ask several students to explain their strategies to the class. Include a variety of strategies, including using skip counting, using partial products, and multiplying the number of rows by the number of pieces of fruit in each row.

After a variety of strategies have been presented, have students evaluate the different methods by asking the following questions:

- “Which strategies were efficient and easy to use? Why?”
- “Which strategies are similar? How are they alike?”
- “What strategy would you use if you were to solve a problem like this again? Why?”

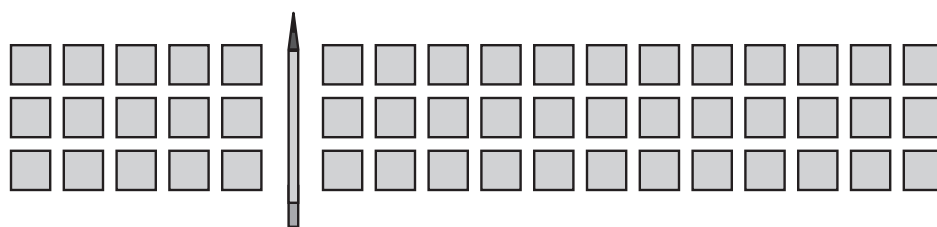
LEARNING CONNECTION 2

Splitting Arrays

MATERIALS

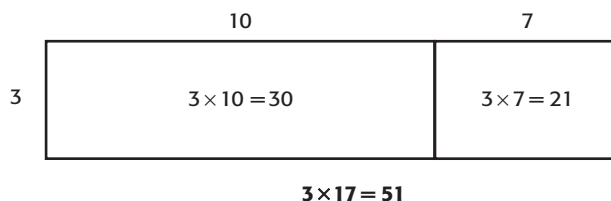
- square tiles (a large number)

Ask pairs of students to arrange tiles to form a 3×17 array. Instruct them to use a pencil to split the array into any two parts. For example, students might split the 3×17 array into a 3×5 array and a 3×12 array.



Next, ask students to explain how they could determine the total number of tiles in the 3×17 array using the smaller arrays. Students might suggest that they could calculate the partial product of each smaller array, and then add the partial products together. Provide time for students to apply this method using the split array that they created.

Ask several pairs of students to draw diagrams on the board, to show how they split their arrays into two parts. Include pairs who worked with partial products that were easy to calculate and then add together, such as $(3 \times 10) + (3 \times 7) = 30 + 21 = 51$, as well as those who worked with less “friendly numbers”, such as $(3 \times 9) + (3 \times 8) = 27 + 24$. Suggest that drawing each tile in the array is a time-consuming task, and encourage students to sketch open arrays instead. Explain that open arrays do not need to be drawn to scale but that the lengths and widths of the rectangles should reflect the size of numbers in the arrays.



Have each pair of students explain how they calculated the total number of tiles by adding the partial products of the smaller arrays.

After a few pairs of students have explained their work, ask:

“Which numbers were “friendly” to work with? Why were these numbers easy to calculate with?”

Repeat the activity using other arrays (e.g., 4×16 or 3×23). Observe whether or not students split arrays into parts that yield numbers that are easy to work with.

LEARNING CONNECTION 3

Some More, Some Less

MATERIALS

- interlocking cubes

Organize students into pairs. Instruct students to make 4 rows of 10 interlocking cubes. Ask them to tell the total number of cubes (40). On the board, record “ $4 \times 10 = 40$ ”.

Ask: “What would you have to do to the rows of cubes you have in order to show 4 groups of 9?” Have students remove one cube from each row, and ask them to tell the number of cubes that are in the four rows. Have students explain their strategies. Emphasize the idea that 4×10 is 40, so 4×9 is 4 less than 40 (i.e., 4 cubes were removed).

Next, have students make 4 rows of 10 cubes again, but this time, ask students to show 4 groups of 11 (by adding a cube to each row). Have students explain their strategies for calculating 4×11 (e.g., since $4 \times 10 = 40$, then 4×11 is 4 more than 40).

Have students use other rows of cubes to derive answers for other multiplication expressions. For example:

- Start with 5×8 , and then calculate 5×9 .
- Start with 5×8 , and then calculate 5×7 .
- Start with 4×5 , and then calculate 4×6 .

Provide practice in using mental computation. Pose pairs of expressions, such as the following, and ask students to explain how they determined the answers:

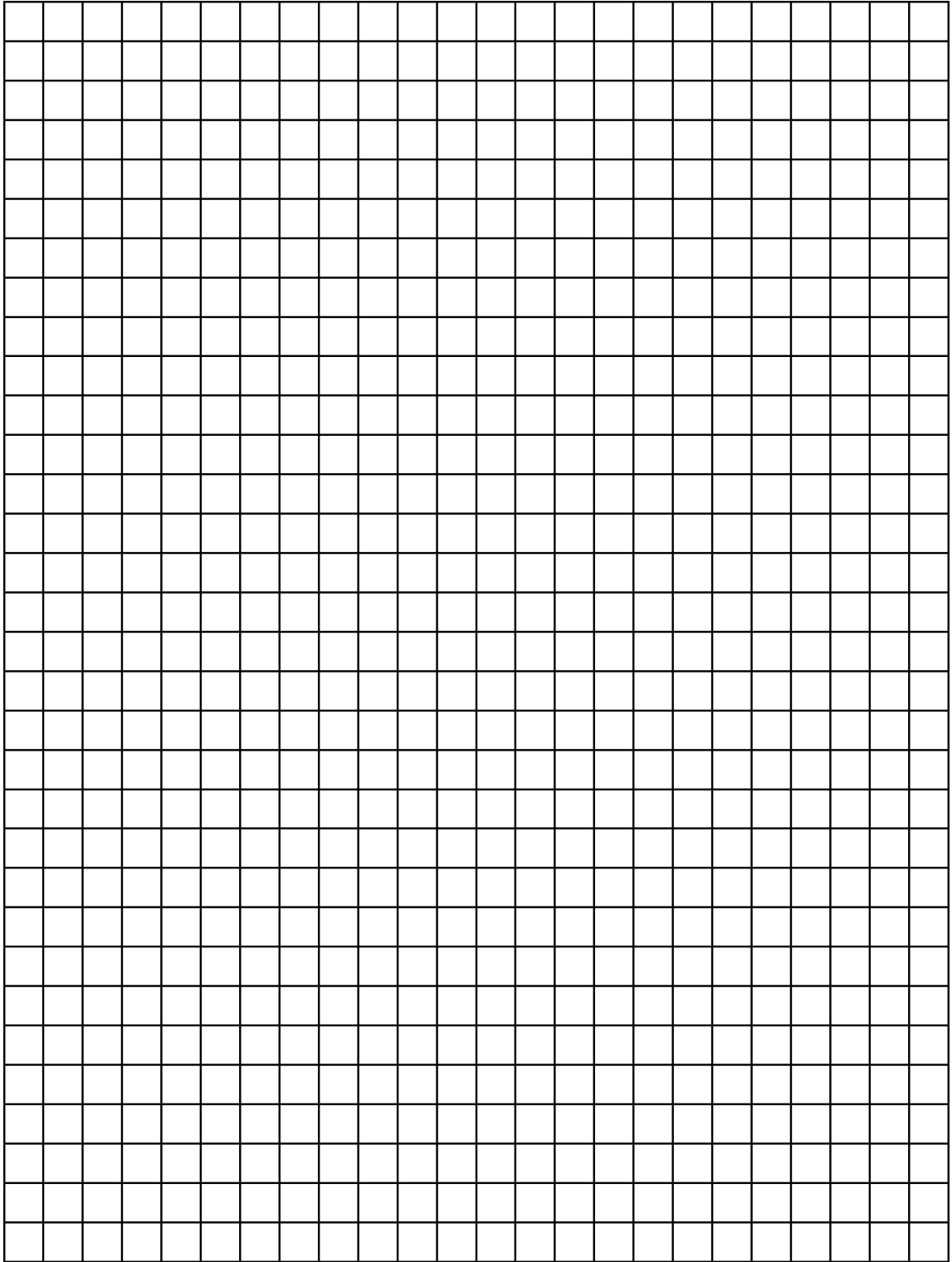
- 4×10 4×9
- 7×10 7×11
- 8×5 8×6
- 8×5 8×4

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts. On the homepage, click “Toolkit”. In the “Numeracy” section, find “Multiplication and Division (4 to 6)”, and then click the number to the right of it.



Grid Paper

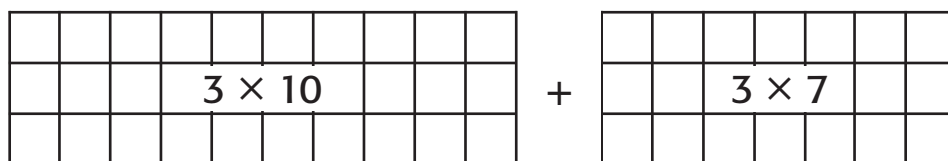
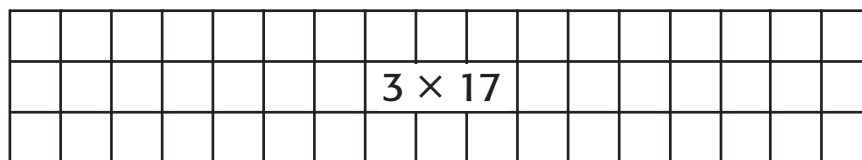
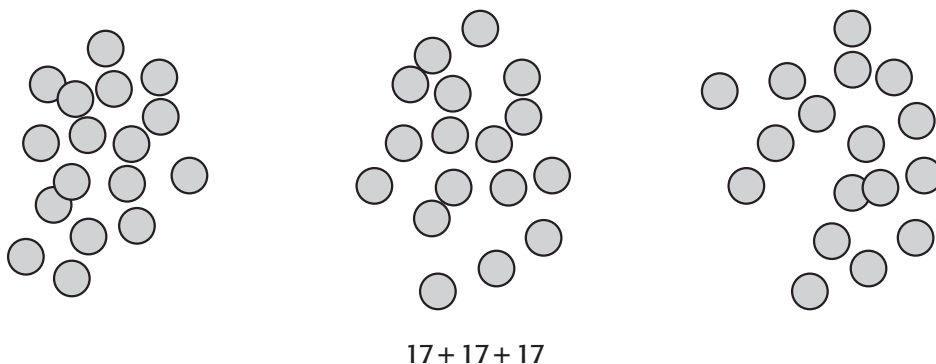


Exploring Multiplication

Dear Parent/Guardian:

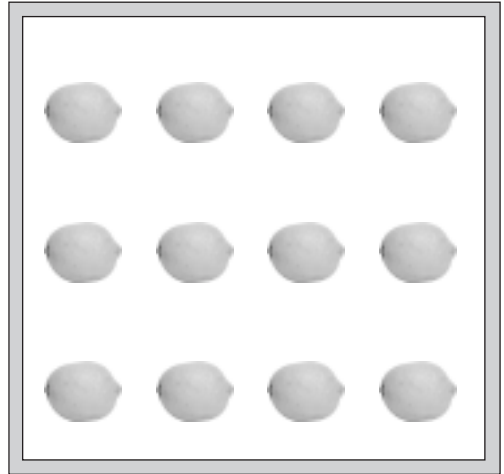
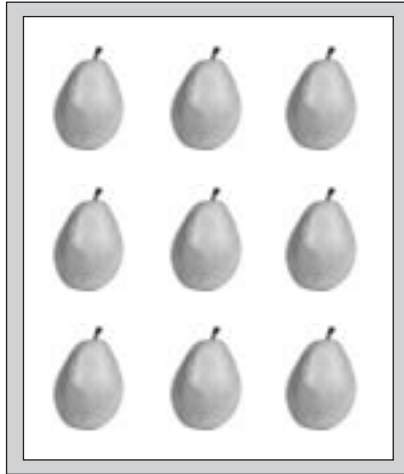
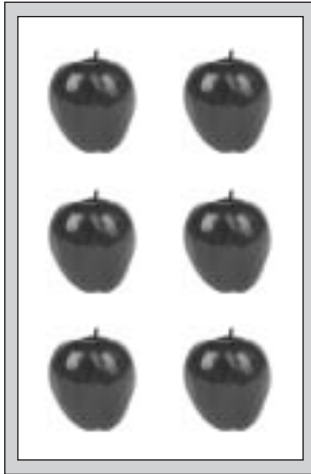
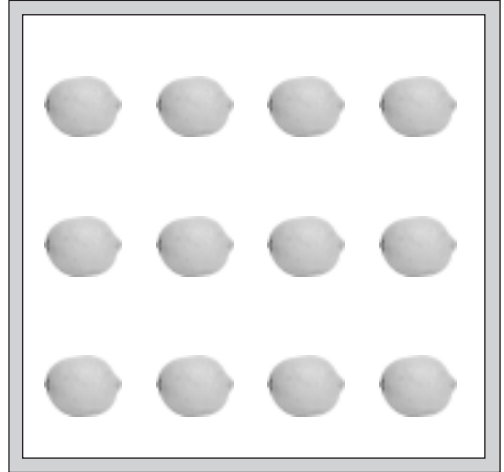
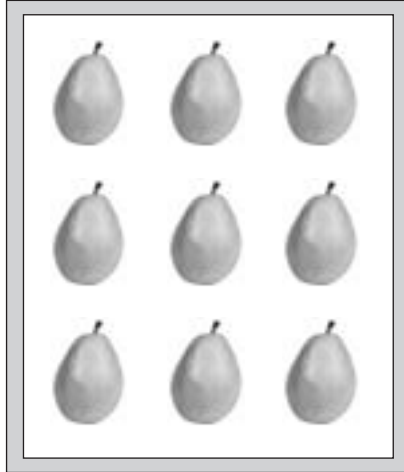
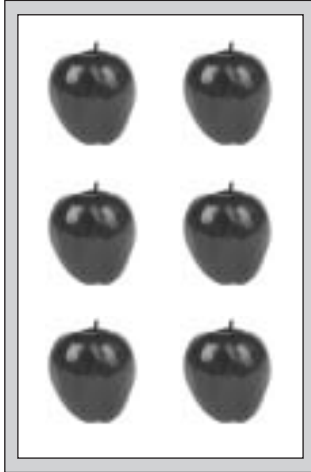
We have been learning about the meaning of multiplication.

Here are three different ways to represent 3×17 . Ask your child to explain each representation and how each representation could be used to find the answer to 3×17 .



Thank you for doing this activity with your child.

How Many Fruits?



Grade 5 Learning Activity

Finding the Cost of a Field Trip

OVERVIEW

In this learning activity, students solve a problem in which they determine the cost of a field trip for 29 students who each pay \$20. The problem-solving experience provides an opportunity for students to explore a variety of multiplication strategies, including the use of the distributive property.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students use a variety of strategies to solve a problem involving the multiplication of a two-digit number by a two-digit number. After solving the problem, students discuss how the distributive property can be used in multiplication.

Relationships: The activity allows students to recognize relationships between operations (e.g., the relationship between repeated addition and multiplication).

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectation**.

Students will:

- multiply two-digit whole numbers by two-digit whole numbers, using estimation, student-generated algorithms, and standard algorithms.

This specific expectation contributes to the development of the following **overall expectation**.

Students will:

- solve problems involving the multiplication and division of multidigit whole numbers, and involving the addition and subtraction of decimal numbers to hundredths, using a variety of strategies.

ABOUT THE LEARNING ACTIVITY

MATERIALS

- sheets of paper (1 per pair of students)
- sheets of chart paper or large sheets of newsprint (1 per pair of students)
- markers (a few per pair of students)
- sheets of paper or math journals (1 per student)
- play money (optional)
- **Mult5.BLM1: 30 Packages!** (1 per student)

TIME:
approximately
two 60-minute
periods

MATH LANGUAGE

- skip counting
- repeated addition
- doubling
- multiplication expression
- open array
- factor
- product
- partial products
- friendly number

INSTRUCTIONAL SEQUENCING

This learning activity provides an introductory exploration of strategies for multiplying a two-digit number by a two-digit number. Before starting this learning activity, students should have an understanding of multiplication concepts (e.g., that multiplication involves combining groups of equal size) and of strategies for multiplying a two-digit number by a one-digit number. Students should also be able to decompose a two-digit whole number into tens and ones (e.g., $29 = 20 + 9$) and should understand how to multiply numbers by multiples of 10.

ABOUT THE MATH

In this learning activity, students solve a multiplication problem using strategies that make sense to them. When students apply their own methods, they develop a deeper understanding of the operation and of the efficiency of different strategies.

This learning activity provides an opportunity for teachers to introduce the use of the distributive property in multiplication: for example, to multiply 29×20 , students might decompose 29 into $20 + 9$, then multiply 20×20 and 9×20 , and then add the partial products ($400 + 180 = 580$).

Using compensation is also an application of the distributive property. With this strategy, students multiply more than is needed, and then remove the “extra” amount. To multiply 29×20 , students might recognize that 29 is close to 30 and multiply 30×20 to get 600. They then subtract 20 (the difference between 30×20 and 29×20) to get 580.

The experience of solving problems using their own methods allows students to apply the distributive property in informal, yet meaningful, ways.

Note: It is not necessary for students to define the “distributive property”, but they should learn how it can be applied to facilitate multiplication.

GETTING STARTED

Describe the following scenario to the class:

“29 students are going on a field trip to a museum. The field trip costs \$20.00 per student. For this fee, each student will receive bus transportation to and from the museum, an entrance ticket to the museum, and a picnic lunch. How much will it cost for 29 students to go on the field trip?”

Divide the class into pairs. Ask students to discuss important information about the problem with their partners. Have students summarize this information. Record the following on the board:

- 29 students
- \$20 per student
- cost for 29 students

WORKING ON IT

Ask students to solve the problem with their partners using a strategy that makes sense to both partners. Provide each pair of students with a sheet of paper on which they can record their work.

As students work on the problem, observe the various strategies they use to solve it. Pose questions to help students think about their strategies and solutions:

- “What strategy are you using to solve the problem?”
- “Why are you using this strategy?”
- “Did you change or modify your strategy? Why?”
- “What materials are you using? How are these materials helpful?”
- “How could you solve the problem in a different way?”
- “How could you represent your strategy so that others will know what you are thinking?”

STRATEGIES STUDENTS MIGHT USE

USING REPEATED ADDITION

Students might record \$20 twenty-nine times, and repeatedly add 20 until they reach a solution.

USING SKIP COUNTING

Students might count by 20’s twenty-nine times.

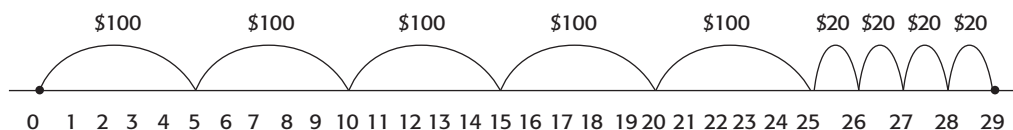
USING GROUPINGS OF \$100

Students might recognize that $5 \times \$20 = \100 and determine the cost for 25 students.

$$\begin{array}{r}
 \$100 \text{ (5 students)} \\
 \$100 \text{ (5 students)} \\
 \$100 \text{ (5 students)} \\
 \$100 \text{ (5 students)} \\
 \underline{\$100 \text{ (5 students)}} \\
 \$500 \text{ (25 students)}
 \end{array}$$

Students would then add the cost for 4 students ($4 \times \$20 = \80) to determine the cost for 29 students ($\$500 + \$80 = \$580$).

USING A NUMBER LINE



(continued)

USING DOUBLING

Students might continue to double the number of students and the related costs.

$$2 \text{ students} \rightarrow 2 \times \$20 = \$40$$

$$4 \text{ students} \rightarrow 2 \times \$40 = \$80$$

$$8 \text{ students} \rightarrow 2 \times \$80 = \$160$$

$$16 \text{ students} \rightarrow 2 \times \$160 = \$320$$

$$32 \text{ students} \rightarrow 2 \times \$320 = \$640$$

Students will realize that 32 students is 3 more than 29 students, and might:

- subtract the cost for 3 students from \$640 ($\$640 - \$60 = \580); or
- combine the costs for 16, 8, 4, and 1 student(s), to calculate the total cost for 29 students ($\$320 + \$160 + \$80 + \$20 = \$580$).

USING A T-CHART

Number of Students	Field Trip Cost
2	\$40
4	\$80
8	\$160
16	\$320
32	\$640
29	\$580

APPLYING THE DISTRIBUTIVE PROPERTY

Some students might apply the distributive property in any of the following ways:

- Decompose 29 into smaller parts, then multiply each part by \$20, and then add the partial products.

Example 1:

$29 \times \$20$ is the same as $(10 + 10 + 9) \times \$20$.

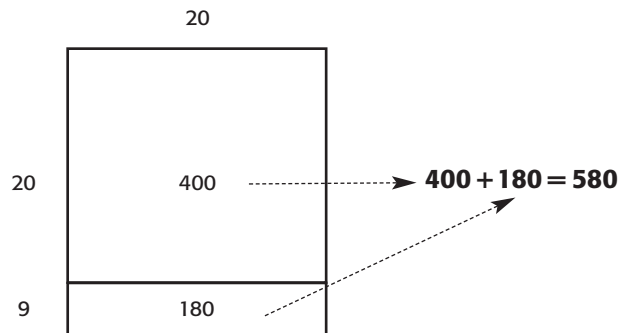
$$(10 \times \$20) + (10 \times \$20) + (9 \times \$20) = \$200 + \$200 + \$180 = \$580$$

Example 2:

$29 \times \$20$ is the same as $(20 + 9) \times \$20$.

$$(20 \times \$20) + (9 \times \$20) = \$400 + \$180 = \$580$$

- Use an open array to model partial products.



- Use a partial product algorithm.

$$\begin{array}{r}
 29 \\
 \times \$20 \\
 \hline
 180 \quad (20 \times 9) \\
 400 \quad (20 \times 20) \\
 \hline
 \$580
 \end{array}$$

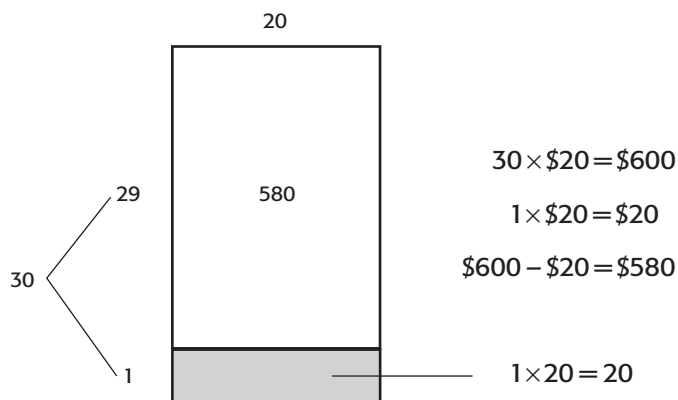
USING COMPENSATION

Some students might recognize that 29 is close to 30. They might determine the total cost of the field trip for 30 students, and then subtract 20 (the extra \$20 fee for one student), to calculate the total cost of the field trip for 29 students.

$$30 \times \$20 = \$600$$

$$\$600 - \$20 = \$580$$

This compensation strategy can be modelled using an open array.



USING A STANDARD ALGORITHM

Students might have learned the procedures in using a standard algorithm. Ask students to explain the meaning of each step in the algorithm. If they are unable to do so, suggest that they use a strategy that they can explain.

$$\begin{array}{r}
 29 \\
 \times \$20 \\
 \hline
 \$580
 \end{array}$$

After students have solved the problem, provide each pair with markers and a sheet of chart paper or a large sheet of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem.

Make a note of groups that might share their strategies and solutions during Reflecting and Connecting. Include groups who used various methods that range in their degree of sophistication (e.g., using repeated addition, using doubling, applying the distributive property).

REFLECTING AND CONNECTING

Reconvene the class. Ask a few groups to share their problem-solving strategies and solution, and post their work. Try to order the presentations so that students observe inefficient strategies (e.g., using repeated addition, using skip counting) first, followed by more efficient methods.

As students explain their work, ask questions that probe their thinking:

- “How did you determine the total cost of the field trip for 29 students?”
- “Why did you use this strategy? How did the numbers in the problem help you choose a strategy?”
- “Was your strategy easy or difficult to use? Why?”
- “Would you use this strategy if you solved another problem like this again? Why or why not?”
- “How would you change your strategy the next time?”
- “How did you record your strategy?”
- “Is your strategy similar to another strategy? Why or why not?”
- “How do you know that your solution is correct?”

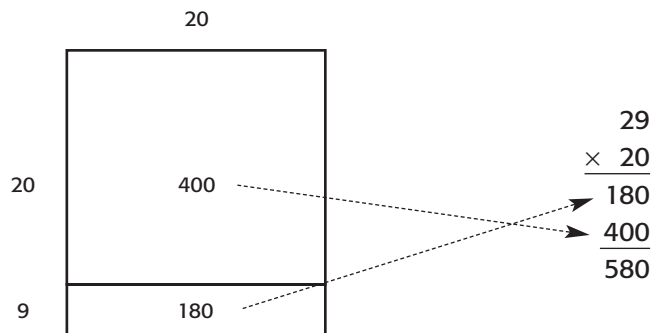
Following the presentations, ask students to observe the work that has been posted, and to consider the efficiency of the various strategies. Ask:

- “Which strategy, in your opinion, is an efficient strategy?”
- “Why is the strategy effective in solving this kind of problem?”
- “How would you explain this strategy to someone who has never used it?”

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Refer to students’ work to emphasize important ideas about the distributive property:

- Two-digit factors can be decomposed into parts to facilitate multiplication (e.g., 29 can be decomposed into 20 and 9, which are friendly numbers that are easy to multiply). Each part is multiplied by the other factor in the multiplication expression ($20 \times 20 = 400$, $9 \times 20 = 180$), and then the partial products are added to calculate the total product ($400 + 180 = 580$).
- An open array shows how factors in a multiplication expression can be decomposed into two or more parts. Partial products are recorded on the open array, and then added to calculate the total product.
- The parts in an open array can be represented in a partial product algorithm.



Provide an opportunity for students to solve a related problem. Explain that along with the 29 students, 7 adults will be going on the field trip to help supervise. The fee for each adult is also \$20 per person. Ask students to give the multiplication expression related to the problem. Record " 36×20 " on the board.

Have students work in pairs. Encourage them to consider the various strategies that have been discussed and to apply a method that will allow them to solve the problem efficiently. After pairs have solved the problem, ask students to independently record a solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Encourage students to use strategies that make sense to them. Recognize that some students may need to rely on simple strategies, such as repeated addition and skip counting, and may not be ready to apply more sophisticated strategies.

Some students may benefit from using play money (e.g., \$20 bills) to represent the problem and to find a strategy. Scaffold the problem by asking:

- "How many \$20 bills will be needed for 1 student?"
- "How many \$20 bills will be needed for 5 students? What will the cost be for 5 students? For 10 students? For 20 students? For 29 students?"

Guide students in using more efficient strategies when you observe that they are ready to do so. Provide opportunities for these students to work with classmates who can demonstrate the use of more efficient strategies.

Challenge students to solve the problem in different ways. For example, if students use an algorithm, ask them to explain how the algorithm works and the meaning of the numbers in the algorithm within the context of the problem.

ASSESSMENT

Observe students as they solve the problem, and assess how well they:

- represent and explain the problem;
- apply an appropriate strategy for solving the problem;
- explain their strategy and solution;
- judge the efficiency of various strategies;
- modify or change strategies to find more efficient ways to solve the problem;
- explain ideas about the distributive property (e.g., that 29×20 can be decomposed into $(20 \times 20) + (9 \times 20)$, and that the partial products, 400 and 180, can be added to determine the final product).

Collect the math journals or sheets of paper on which students recorded their strategies for determining the cost of a trip for 29 students and 7 adults. Observe students' work to determine how well they apply an efficient strategy for solving the multiplication problem.

HOME CONNECTION

Send home **Mult5.BLM1: 30 Packages!** In this Home Connection activity, students are asked to find packages that contain between 10 and 50 items, and then calculate the number of items that would be in 30 packages.

LEARNING CONNECTION 1

Applying the Distributive Property

MATERIALS

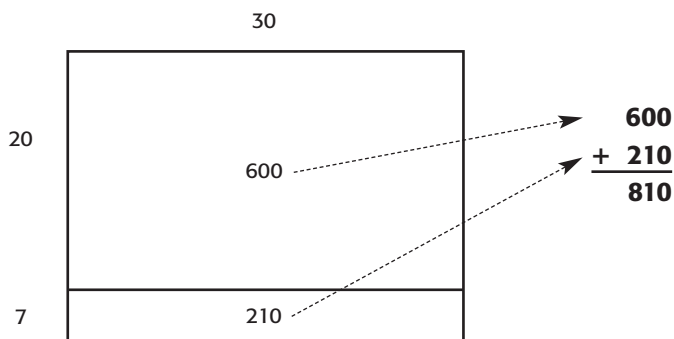
- sheets of paper (1 per student)

Provide opportunities for students to use the distributive property to simplify multiplication. Record " 27×30 " on the board, and challenge students to calculate the product mentally. (Allow students to keep track of some of their calculations on paper, but encourage them to do most of the calculations in their head.) Have students explain their strategies.

Discuss the steps in the following strategy:

- 27 can be decomposed into 20 and 7.
- Each part can be multiplied by 30 ($20 \times 30 = 600$, $7 \times 30 = 210$).
- The partial products can be added to calculate the total product ($600 + 210 = 810$).

Illustrate the strategy using an open array.



Provide other multiplication expressions for students to calculate mentally:

- 43×30
- 58×40
- 62×50

Have students draw open arrays on the board to represent the calculation of each expression using the distributive property.

LEARNING CONNECTION 2

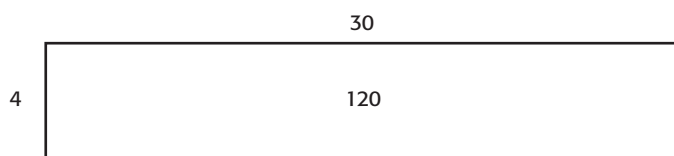
What Would the Array Look Like?

MATERIALS

- sheets of paper (1 per student)

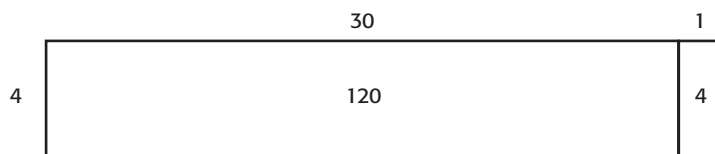
Discuss with students how it is impractical to use square tiles to create arrays that involve larger numbers (e.g., to show a 4×30 array). Have students suggest other ways to show an array with large numbers (e.g., using grid paper, using an open array). Talk about the advantages of using open arrays (e.g., they are easy to draw; they can represent large numbers; they are uncluttered).

On the board, record " 4×30 ". Ask students to imagine what the open array would look like, and then have them draw an open array on a sheet of paper. Draw an example on the board, and explain that an open array does not need to be drawn to scale but that the length and width of the rectangle should reflect the size of the numbers in the array.



Ask students to give the product of 4×30 , and record "120" on the array.

Next, record " 4×31 " on the board, and ask students to explain how the open array would compare with the one for 4×30 . Instruct students to add a section to their 4×30 array to show 4×31 .



Ask students to explain how the open array can help them calculate 4×31 (e.g., $3 \times 40 = 120$ and $4 \times 1 = 4$, so 4×31 is $120 + 4$, or 124).

Repeat the activity by having students draw open arrays for the following pairs of multiplication expressions:

- 3×20 3×22
- 5×40 5×43
- 7×30 7×35

For each pair, ask students to explain how the open array for the first multiplication expression can help them calculate the product for the second expression.

LEARNING CONNECTION 3

Exploring the Commutative Property of Multiplication

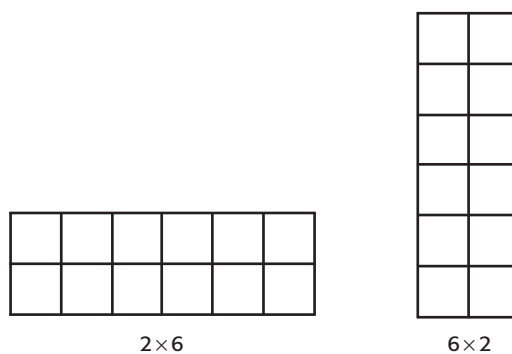
MATERIALS

- square tiles (12 per pair of students)
- **Mult5.BLM2: Large Grid Paper** (a few sheets per pair of students)
- scissors (1 pair per pair of students)
- sheets of paper (1 per student)

Provide an opportunity for students to represent and discuss the commutative property of multiplication. Give 12 square tiles to each pair of students, and ask students to create as many different arrays as possible. Instruct them to make paper cut-outs of each array using **Mult5.BLM2: Large Grid Paper**.

After students have finished cutting out the different arrays, ask them to discuss observations about the arrays with their partners. Provide a few minutes for discussion, and then reconvene the whole class. Invite students to share their observations with the large group.

On the board, post a 2×6 array and a 6×2 array, and record the multiplication expression next to each.



Have students compare the two arrays and discuss:

- Both arrays involve the same factors (2 and 6), but their order is different in each array.
- The 2×6 and the 6×2 arrays are congruent (i.e., they both involve the same arrangement of squares).
- One array is a rotation of the other.

Next, post a 3×4 array and a 4×3 array, and record the multiplication expression next to each. Again, have students share their observations about the two arrays.

Hold a similar discussion about the 1×12 and the 12×1 arrays.

Ask:

- “What do the pairs of arrays show about multiplication?” (The factors in a multiplication expression can be placed in any order.)
- “How could you determine the answer to 9×3 if you know the answer to 3×9 ?”

Provide each student with a sheet of paper. Challenge them to prove on paper that 4×8 has the same product as 8×4 . Observe students' work, to assess how well they understand the commutative property of multiplication.

Note: It is not necessary for students to define the "commutative property", but they should learn how it can be applied to facilitate multiplication.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Multiplication and Division (4 to 6)", and then click the number to the right of it.



30 Packages!

Dear Parent/Guardian:

We have been learning about multiplication.

Have your child work on the following activity. Ask your child to explain his or her work.

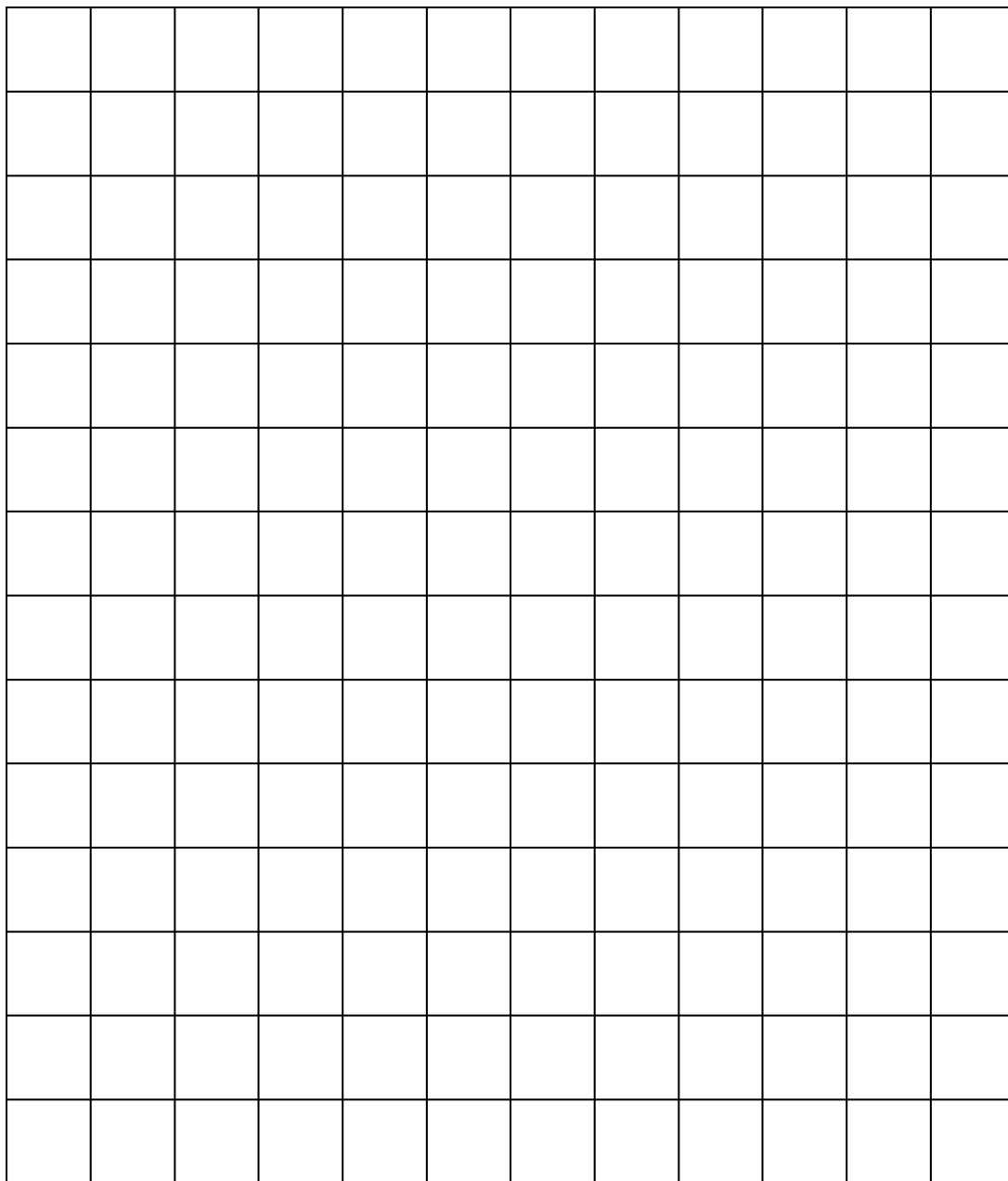
Find packages in your home that contain between 10 and 50 items (for example, bottles of water in a case, eggs in a carton, tea bags in a box). Find the total number of items in the package.

Next, imagine that you have 30 packages. Determine the total number of items in 30 packages. Show your work in the space below.

Explain your work to a family member.

Thank you for doing this activity with your child.

Large Grid Paper



Grade 6 Learning Activity

Shopping for Puppy Food

OVERVIEW

In this learning activity, students use a variety of multiplicative strategies (e.g., using repeated addition, using doubling, using proportional reasoning) to calculate and compare the costs of 24 cans of puppy food at three different stores.

BIG IDEAS

This learning activity focuses on the following big ideas:

Operational sense: Students use a variety of strategies to solve a problem involving multiplicative reasoning and discuss the efficiency of various strategies.

Relationships: Students compare costs expressed as decimal numbers.

Proportional reasoning: The learning activity provides an opportunity for students to apply proportional reasoning to determine the cost of 24 cans of puppy food.

CURRICULUM EXPECTATIONS

This learning activity addresses the following **specific expectations**.

Students will:

- represent, compare, and order whole numbers and decimal numbers from 0.001 to 1 000 000, using a variety of tools (e.g., number lines with appropriate increments, base ten materials for decimals);
- multiply and divide decimal numbers to tenths by whole numbers, using concrete materials, estimation, algorithms, and calculators (e.g., calculate 4×1.4 using base ten materials; calculate $5.6 \div 4$ using base ten materials);
- represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation.

These specific expectations contribute to the development of the following **overall expectations**.

Students will:

- read, represent, compare, and order whole numbers to 1 000 000, decimal numbers to thousandths, proper and improper fractions, and mixed numbers;
- solve problems involving the multiplication and division of whole numbers, and the addition and subtraction of decimal numbers to thousandths, using a variety of strategies;
- demonstrate an understanding of relationships involving percent, ratio, and unit rate.

ABOUT THE LEARNING ACTIVITY

TIME:
approximately
60 minutes

MATERIALS

- sheets of paper (1 per group of 2 or 3 students)
- sheets of chart paper or large sheets of newsprint (1 per group of 2 or 3 students)
- markers (a few per group of 2 or 3 students)
- sheets of paper or math journals (1 per student)
- **Mult6.BLM1: 2 for ..., 3 for ...** (1 per student)

MATH LANGUAGE

- repeated addition
- doubling
- factor
- product
- partial products
- rate

**INSTRUCTIONAL
GROUPING:**
groups of 2 or 3

INSTRUCTIONAL SEQUENCING

Prior to this learning activity, students should have had opportunities to add decimal numbers to hundredths (e.g., money amounts), and to represent simple multiplicative relationships involving rates (e.g., “If a box contains 6 markers, then 6 boxes contain 36 markers.”).

ABOUT THE MATH

An understanding of multiplicative situations is critical in the development of students’ mathematical thinking. This understanding allows students to:

- express relationships between quantities (e.g., “There is 3 times as much snow on the ground today as yesterday.”);
- solve problems involving rates (e.g., “If 2 books cost \$7.25, then 4 books cost \$14.50.”);
- determine equivalent fractions (e.g., “If young children sleep about $\frac{1}{3}$ of a day, then they sleep about 8 hours or $\frac{8}{24}$ of a day.”);
- reason proportionally (e.g., “A DJ plays 3 fast songs for every slow song, so if the DJ plays 4 slow songs, then she plays 12 fast songs.”).

This learning activity provides an opportunity for students to solve a problem involving multiplicative relationships. The experience of using various informal strategies (e.g., using repeated addition, using doubling, using ratio tables) allows students to comprehend multiplicative situations, and prepares them for learning more formal strategies in subsequent grades.

GETTING STARTED

Explain the following situation to the class:

“My friend called me last evening because he was very excited about getting a new puppy. My friend explained that the puppy needs special food to help it to grow up to be a healthy dog. He told me that there are three stores in his neighbourhood that sell the special puppy food.”

On the board, record the store names and prices for the puppy food:

- Pat’s Pet Emporium: \$0.80 per can
- Pet-o-rama: \$9.40 for a dozen cans
- Petmania: \$2.55 for three cans

Pose the problem: “My friend wants to buy 24 cans of puppy food. How much will he pay for the puppy food at each store? At which store will he get the best price?”

Ensure that students understand the problem. Ask:

- “What do you need to find out?”
- “What information will you need to use to solve the problem?”

WORKING ON IT

Organize students into groups of two or three. Encourage them to work collaboratively to solve the problem. Provide each group with a sheet of paper on which students can record their work.

Observe students as they solve the problem. Ask questions that help students think about their problem-solving strategies and solutions:

- “How are you solving the problem?”
- “What part of this problem is easy for you to solve? What is difficult?”
- “How can you determine the cost of 24 cans at Pat’s Pet Emporium? Pet-o-rama? Petmania?”
- “What other strategies can you use to determine the cost of 24 cans at each store?”
- “How can you record your solution so that others will understand how you solved the problem?”

STRATEGIES STUDENTS MIGHT USE

USING DOUBLING

Many students will double \$9.40 (the cost of a dozen cans) to calculate the cost of 24 cans at Pet-o-rama (\$18.80).

To determine the cost of 24 cans at Pat’s Pet Emporium and at Petmania, students might use a variety of strategies.

(In the following examples, decimal points have been included in the calculations. It is also acceptable for students to perform the calculations using whole numbers, and then add dollar signs and decimal points to the results to indicate monetary amounts.)

USING REPEATED ADDITION

Students might repeatedly add the cost of single cans until they determine the cost of 24 cans (e.g., adding 0.80 twenty-four times to calculate the cost of 24 cans at Pat’s Pet Emporium).

$$\begin{array}{r}
 0.80 \\
 + 0.80 \\
 \hline
 1.60 \\
 + 0.80 \\
 \hline
 2.40 \\
 + 0.80 \\
 \hline
 3.20 \\
 + 0.80 \\
 \hline
 \text{and so on}
 \end{array}$$

USING DOUBLING

Students might repeatedly double the number of cans and their costs. For example, for Pat's Pet Emporium:

$$0.80 + 0.80 = 1.60 \text{ (2 cans)}$$

$$1.60 + 1.60 = 3.20 \text{ (4 cans)}$$

$$3.20 + 3.20 = 6.40 \text{ (8 cans)}$$

$$6.40 + 6.40 = 12.80 \text{ (16 cans)}$$

$$\$12.80 \text{ (the cost of 16 cans)} + \$6.40 \text{ (the cost of 8 cans)} = \$19.20 \text{ (the cost of 24 cans)}$$

USING A RATIO TABLE

Students might use a ratio table to generate the cost of 24 cans. For example, to determine the cost of 24 cans at Petmania, students might double the number of cans and the costs of the cans until they determine the cost of 24 cans.

Number of cans	3	6	12	24
Cost	\$2.55	\$5.10	\$10.20	\$20.40

USING PARTIAL PRODUCTS

To calculate the cost of 24 cans at Pat's Pet Emporium, students might decompose 24 into 10, 10, and 4, then multiply each number by \$0.80, and then add the partial products.

$$\text{Cost of 10 cans} \rightarrow 10 \times \$0.80 = \$8.00$$

$$\text{Cost of 4 cans} \rightarrow 4 \times \$0.80 = \$3.20$$

$$\text{Cost of 24 cans} \rightarrow \$8.00 + \$8.00 + \$3.20 = \$19.20$$

(continued)

USING A MULTIPLICATION ALGORITHM

Students might use an algorithm to calculate the cost.

$$\begin{array}{r} 24 \\ \times 0.8 \\ \hline 19.2 \end{array}$$

Cost of 24 cans = \$19.20

USING PROPORTIONAL REASONING

To determine the cost of 24 cans at Petmania, students might recognize that 3 is a factor of 24 ($3 \times 8 = 24$) and use this multiplicative relationship to reason proportionally; that is, multiply \$2.55 (the cost of 3 cans) by 8 to calculate the cost of 24 cans.

When students have solved the problem, provide each group with markers and a sheet of chart paper or large sheets of newsprint. Ask students to record their strategies and solutions on the paper, and to clearly demonstrate how they solved the problem.

Make a note of the various strategies used by students, and consider which groups might present their strategies during Reflecting and Connecting. Aim to include a variety of strategies that range in their degree of efficiency (e.g., using repeated addition, using doubling, using partial products, using proportional reasoning).

REFLECTING AND CONNECTING

Reconvene the class after the students have solved the problem. Begin a discussion by asking general questions about the problem-solving experience:

- "How did your group decide how to solve this problem?"
- "What was easy about solving this problem?"
- "What was difficult about solving the problem?"

Have a few groups present their strategies for determining the cost of 24 cans at the three pet stores, and for comparing the prices.

As students explain their work, ask questions that probe their thinking, and encourage them to explain their strategies:

- "How did you determine the cost of 24 cans at each store?"
- "Why did you use this strategy?"
- "What worked well with this strategy? What did not work well?"
- "Would you use this strategy if you solved another problem like this again? Why or why not?"
- "How would you change your strategy the next time?"
- "How did you record your strategy?"
- "Which store offers the best price? How do you know?"

Following the presentations, encourage students to consider the efficiency of the various strategies that have been presented. Ask:

- “In your opinion, which strategy worked well?”
- “Why is the strategy effective in solving this kind of problem?”
- “How would you explain this strategy to someone who has never used it?”

Avoid making comments that suggest that some strategies are better than others – students need to determine for themselves which strategies are meaningful and efficient, and which ones they can make sense of and use.

Pose the following problem:

“A flyer for a pet store advertises a special – 4 cans of puppy food for \$2.90. What is the cost of 24 cans?”

Have students work independently to solve the problem. Encourage them to think back to the different strategies presented by classmates, and to use an efficient strategy that makes sense to them. Have students show their strategies and solution on a sheet of paper or in their math journals.

ADAPTATIONS/EXTENSIONS

Some students may benefit from solving a version of the problem that involves simpler numbers (e.g., determining the best buy given \$1.00 per can, \$9.50 for 10 cans, \$2.20 for 2 cans).

For students who require a greater challenge, extend the problem by having them determine the amount of money saved if a person buys 72 cans at Pet-o-rama rather than at Petmania.

ASSESSMENT

Observe students as they solve the problem:

- How efficient are students’ strategies for determining the cost of 24 cans at each pet store?
- How well do students apply proportional reasoning?
- How accurate are students’ calculations?
- Are students able to compare prices?
- How well do students explain their strategies and solutions?
- Are students able to judge the efficiency of various strategies?

Examine students’ solutions for the problem posed at the conclusion of Reflecting and Connecting. Assess how well students selected and applied efficient strategies to solve that problem.

HOME CONNECTION

Send home **Mult6.BLM1: 2 for ..., 3 for** In this Home Connection activity, parents and students discuss the prices of grocery store items that are sold at rates, such as “3 for \$1.49”.

LEARNING CONNECTION 1

How Much Larger Is That Letter?

MATERIALS

- **Mult6.BLM2: How Much Larger Is That Letter?** (1 per student)

Provide each student with a copy of **Mult6.BLM2: How Much Larger Is That Letter?** and have them complete the three activities described. These activities help students to connect ideas about scaling and proportions.

LEARNING CONNECTION 2

Ratios and Rates are Everywhere!

MATERIALS

- newspapers, magazines, store flyers (brought to class by students)
- scissors (1 pair per student)
- large sheets of paper (1 per group of 3 or 4 students)
- glue

Discuss the meaning of *ratio* and *rate*. For example, a *ratio* is a comparison of similar types of things, as in “3 cars to 4 trucks” (both cars and trucks are vehicles); whereas a *rate* involves a comparison of two items with different units, as in “60 kilometres per hour” or “6 cans for \$2.99”. Have students give examples of ratios and rates.

Arrange students in groups of three or four. Instruct them to create a collage by cutting out examples of ratios and rates in newspapers, magazines, and store flyers, and gluing them onto a large sheet of paper. Encourage students to organize their examples in some way, such as according to “ratios” and “rates”, by kinds of items, or in the ways used to express ratios and rates (e.g., 3 for \$0.99, 3/\$0.99).

Have groups present their examples. Discuss how rates and ratios are used, and the various ways to express them.

LEARNING CONNECTION 3

Estimating the Cost of Breakfast

MATERIALS

- **Mult6.BLM3: Estimating the Cost of Breakfast** (1 per student)

Arrange students in pairs. Provide each pair with a copy of **Mult6.BLM3: Estimating the Cost of Breakfast**.

Discuss the problem, and explain that students are to estimate Lenore’s cost for all the breakfast foods. Encourage students to use estimation strategies that make sense to them.

Have pairs of students present their findings to the class. Discuss the different estimation strategies used by students.

LEARNING CONNECTION 4

Using the Associative Property to Simplify Multiplication

On the board, record " $4 \times 7 \times 5$ ". Ask students to mentally calculate the answer by multiplying 4×7 first, and then multiplying 28×5 .

Next, record " $4 \times 5 \times 7$ " on the board, and again, have students multiply the factors from left to right ($4 \times 5 = 20$, $20 \times 7 = 140$).

Ask students to compare the multiplication expressions and the answers. Emphasize the idea that the factors in both expressions are the same but presented in a different order, and that the product is the same for both expressions.

Ask students to explain which expression was easier to calculate. Students might comment that the multiplications in the second expression (4×5 and 20×7) were easier to perform than the multiplications in the first expression (4×7 and 28×5).

Have students calculate other pairs of expressions:

$$\begin{array}{ll} 2 \times 8 \times 5 & 2 \times 5 \times 8 \\ 4 \times 9 \times 5 & 4 \times 5 \times 9 \\ 5 \times 7 \times 8 & 5 \times 8 \times 7 \end{array}$$

Next, have students propose a strategy for multiplying three or more factors (e.g., the order of the factors can be changed to facilitate multiplication because changing the order of the factors does not change the product).

Have students apply the strategy to calculate other multiplication expressions, such as the following:

- $8 \times 6 \times 5$
- $3 \times 5 \times 8 \times 2$
- $5 \times 9 \times 6$
- $4 \times 8 \times 5 \times 3$

LEARNING CONNECTION 5

Halving and Doubling

MATERIALS

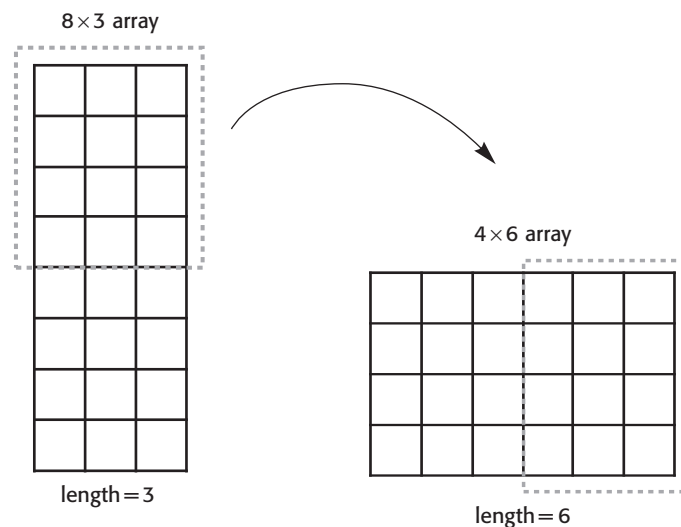
- square tiles (24 for each pair of students)
- sheets of paper (1 per student)

Record the following table on the board or on chart paper.

Array	Width	Length
8×3		
4×6		
2×12		
1×24		

Arrange students in pairs. Provide each pair with 24 square tiles, a sheet of paper, and a pencil. Have pairs copy the table onto their paper and then use the tiles to create the arrays listed. Instruct students to record the width and length of each array on their table.

Talk about the activity after students have completed their tables. Discuss how the number of tiles remains the same for each array. Have students explain how they created each new array from the previous one. For example, students may have slid the top half or bottom half of an array to create a new array.



Ask students to describe patterns in the table. Emphasize the idea that the width of each array is half the width of the previous array, while the length is double the length of the previous array.

Repeat the activity by having students create arrays for 8×2 , 4×4 , 2×8 , and 1×16 , and have them record their findings in a table. Discuss how the width is halved and the length is doubled with each new array.

Ask students how they might use a halving-and-doubling strategy to calculate 4×17 . Students might suggest that they could halve 4 and double 17 to create 2×34 . Discuss how 2×34 is easier to calculate mentally than 4×17 .

Have students practise halving and doubling with other multiplication expressions, such as 16×5 , 24×5 , 12×25 , 18×25 , 18×50 , and 42×50 . Discuss situations in which the strategy is useful for performing mental calculations.

Extend the activity by having students investigate whether doubling first and then halving is a workable strategy. Have them propose multiplication expressions for which a doubling-and-halving strategy would be useful.

eWORKSHOP CONNECTION

Visit www.eworkshop.on.ca for other instructional activities that focus on multiplication concepts. On the homepage, click "Toolkit". In the "Numeracy" section, find "Multiplication and Division (4 to 6)", and then click the number to the right of it.



2 for ..., 3 for ...

Dear Parent/Guardian,

We are learning about multiplication.

Grocery store items are often sold in quantities of 2 or 3 (for example, 2 cans for \$1.99, 3 bottles for \$3.49).

Look through a grocery store flyer with your child, and find examples of food that are sold as "2 for ...", "3 for ...", "6 for ...", and so on.

Select an item and have your child use a calculator to find the cost of multiple items. For example, if the price of tomato soup is 3 cans for \$1.39, you might use a table to record the price of 3, 6, 9, 12, and 15 cans.

Cans	3	6	9	12	15
Price	\$1.39	\$2.78	\$4.17	\$5.56	\$6.95

Next, select a different item with your child. Without your child watching, use a calculator to determine the cost of several of the items. Tell your child the cost of several items, and have him or her estimate the number of items. For example, if containers of yogurt are 2 for \$4.89, you might calculate the cost of 12 containers, and ask: "How many containers could I buy for \$29.34?" Have your child use a calculator to check his or her estimate.

Thank you for doing this activity with your child.

How Much Larger Is That Letter?

- Find a way to determine how much larger the second letter in each row is than the first letter. Explain your method to a partner.



- Choose a letter and print it on a piece of paper. Next, print a second letter that is proportionally larger. Remember to increase both the height and the width of the letter by the same factor. For example, if you quadruple the height, you must also quadruple the width.
- Ask a partner to figure out how much larger your second letter is than your first letter. Have your partner explain his or her thinking.

Estimating the Cost of Breakfast

Lenore is a caterer. To help her plan a breakfast for 120 people, she made a table that shows the food she will serve, the amount of food required per person, and the price she needs to pay for each kind of food.

Food	Amount per Person	Price
orange juice	1 juice box	pack of 3 boxes/\$1.29
eggs	2 eggs	\$1.49 per dozen
croissants	1 croissant	6/\$2.49
yogurt	1 container	pack of 8 containers/\$4.89

Estimate how much Lenore needs to pay for all the food.

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