Learning Geometric Concepts through Ceramic Tile Design

CLARE V. BELL

SYMMETRY AND GEOMETRIC PATTERNS are commonly used in the creation of designs that symbolize and contribute to the definition of culture. Native American weaving and pottery designs, Mexican tiles, and Islamic religious art are forms of cultural representation that rely heavily on a repetition of geometric figures and symmetry. These items are used as examples of geometric art for the lessons in this article (see **fig. 1**).

Many designs that have common characteristics within these cultures can be recreated starting with a circle inscribed within a square. The construction of regular polygons within a circle can launch the discovery of the properties of geometric shapes, the relationships among them, and the use of geometry in artistic expression.

The following sequence of lessons is designed to engage middle school students in the use of two-dimensional geometry by creating individually designed and colorfully glazed ceramic tiles. Each piece of student artwork is a lasting product that has personal meaning. The story of each tile and its creation produce multidimensional results—the understanding of the content of the mathematics, the process of its creation, and the significance of the design itself. Additionally, and possibly the foremost benefit for mathematics education, designing and creating the tiles puts a student in the role of being a practicing mathematician. Mathematics is used to examine and gain insight into the art of other cultures and to create an individual and unique design.

As students progress through the lessons, they learn about lines and angles as well as the properties of circles, triangles, quadrilaterals, and regular and nonregular polygons. The use of constructions to explore properties of geometric figures allows students to experience the properties informally. As they build on this informal knowledge, students increase their ability to communicate their understanding of the relationships among angles, sides, and shapes of geometric figures. Through explorations, students develop more precise descriptions and classifications, thus increasing their ability to create and critique inductive and deductive arguments concerning geometric ideas and relationships (NCTM 2000).

Geometric Tile Design: A Series of Lessons

FOUR COPIES OF A SQUARE MASTER PAGE (FOUR, four-inch squares per page), a pencil, a ruler, a compass, and a protractor are required for each student in lessons 1, 2, and 3. Colored pencils are needed in lesson 3, and six-inch square bisque tiles, paintbrushes (no. 2 to no. 4), and underglaze are needed for lesson 4.


Mexican wall tile courtesy of the Texas Tile Center

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Ninth-century Iraqi bowl. Photograph © 2003 Museum Associates/ Los Angeles County Museum of Art (LACMA), the Nasli M. Heeramaneck Collection, gift of Joan Palevsky. All rights reserved.


Native American seed jar, by Nona Naha, courtesy of Pueblo Arts, www.pueblo arts.com/ Hopi/hopi.htm. Photograph by Kathi Ouellet. Used with permission. All rights reserved.

Fig. 1 Examples of artwork that are representative of three different cultures

Lesson 1

STUDENTS IN GRADES 3–5 SHOULD HAVE HAD opportunities to develop the ability to clearly describe the properties of geometric shapes (NCTM 2000). By constructing and exploring regular polygons in this lesson, middle school students can build

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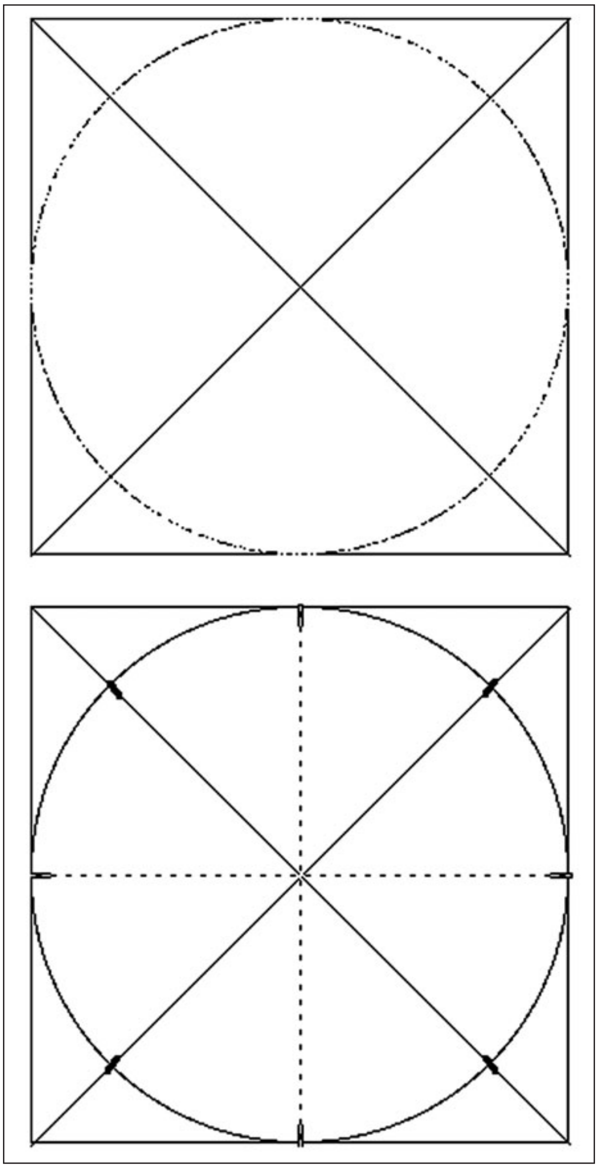


Fig. 2 Illustrations of finding the center point of the square, inscribing a circle, and dividing the circle into equal arcs

on that foundation. Learning how to create accurate polygons does not require extensive mathematical knowledge, but once the polygons are drawn, mathematical lessons “pop up from every corner” (Hardaker 1994–2003). Through mathematical discussion orchestrated by the teacher, students grow in their ability to find and explain relationships between figures and to justify geometric thinking.

Activities and suggestions for discussion and exploration

1. Using the square master page, a ruler, and a pencil, draw the diagonals of the square. The intersection of these diagonals is the center point from which a circle is inscribed in the square using a compass (see **fig. 2**). Throughout this initial construction and those that follow, special attention should be given to eliciting and clarifying definitions, developing accurate terminology, and noting properties of geometric figures. For instance, this lesson can generate the definitions of *square, diagonal, circle, center of circle,* and *radius.*

2. After constructing the circle, divide it into a number of equal-sized arcs. For example, to create an octagon within the circle, divide 360 degrees by 8, resulting in 45 degrees. Draw one radius of the circle. From the radius, use a protractor to measure off 45 degrees successively to divide the circle into equal arcs. Lines should then be drawn from the center point of the circle to the points marked on the circle, creating radii (see **fig. 2**). Finally, adjacent points on the circle are connected to form a regular polygon.

3. Follow this process repeatedly to create other regular polygons. The circles can be divided into three, four, five, six, eight, nine, ten, or twelve arcs fairly easily; then the adjacent dividing points can be connected to form a triangle, square, pentagon, hexagon, octagon, nonagon, decagon, and dodecagon (see **fig. 3**). Ask students to note the numbers of sides and angles and their relative sizes.

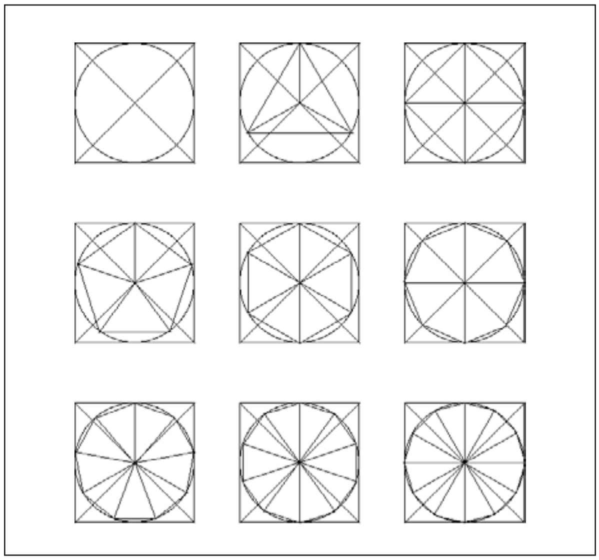


Fig. 3 Regular polygons inscribed within circles

Exploration 1: Students have already calculated the measurement of the angles formed by radii of the circles in step 2, above. With the additional knowledge that the sum of the measurements of the interior angles of any triangle equals 180 degrees, they can deduce the measurements of the interior angles of the regular polygons. The teacher might pose these questions to facilitate students’ interaction with this relationship:

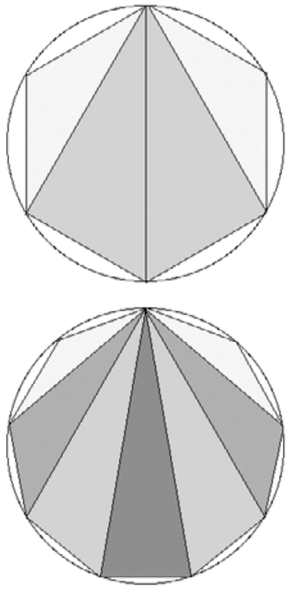


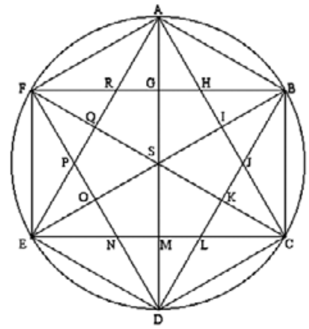
Fig. 4 Pairs of congruent triangles formed by the diagonals drawn from a single vertex

- What do you notice about the triangles that are formed by the radii and the sides of the polygon?
- Which sides of the triangles are congruent?
- Are there congruent angles? How do you know?
- What are the measurements of the angles?
- Can you prove that?
- From what you know about the triangles, calculate the measurement of the interior angles of the polygon. What sequence of steps did you take to find the angle measurement?

Exploration 2: The measurements of the interior angles of regular polygons can also be deduced by creating the diagonals from one vertex within each polygon (see **fig. 4**). These questions can be explored:

- How many triangles are created when you draw all the diagonals from just one vertex of the polygon?
- What is the sum of the measurements of the interior angles of each triangle?
- Are there any angles in the triangles that are not shared by the polygon?
- Find the sum of the interior angles of all the triangles. How many angles are in the polygon?
- Find the measurement of each angle of the polygon by using what you know about the triangles. The goal is to discover that the total of the angle measurements for all the triangles divided by the number of angles of the polygon will result in the measurement of each interior angle of the polygon.

Either exploration illustrates that congruent triangles are formed. In exploration 2, all the diagonals created from only *one* vertex of a regular polygon will divide the polygon into pairs of congruent triangles in regular polygons with an even number of sides, and pairs of congruent triangles with an additional isosceles triangle in regular polygons with an odd number of sides (see **fig. 4**). Record findings in a table (see **table 1**).



Congruent Triangles	Congruent Quadrilaterals	Congruent Pentagons
△ *ABH*		
△ *BCJ*		
△ *CDL*		
△ *DEN*		
△ *EFP*		
△ *FAR*		
△ *ARG*		
△ *AHG*		
△ *BHI*		
△ *BJI*		
△ *CJK*		
△ *CLK*, and so on		

Fig. 5 Example sheet for recording polygons within the hexagon

Lesson 2

BY CONSTRUCTING A REGULAR HEXAGON, STUDENTS explore properties of—and classify and express relationships between—triangles, quadrilaterals, and pentagons. Students find additional polygons formed by the diagonals of the original hexagon. The ability to visualize a variety of shapes within the diagonals enables the creation of new designs and patterns.

Activities and suggestions for discussion and exploration

1. Follow directions for the first step in lesson 1. After constructing a circle, students will divide it into six equal-sized arcs as described previously.

Adjacent points on the circle are then connected to form a regular hexagon.

2. The vertices of the hexagon should be labeled with capital letters. From each vertex, create all possible diagonals. (Record the results for future combinatorial work with counting diagonals.) Each new intersection will also be labeled, as suggested in **figure 5.**

3. Challenge students to find as many triangles as possible. Guide students to use the labeled vertices to identify the triangles and to use a systematic approach to list congruent triangles, as shown in **figure 5.** Discuss the characteristics of different triangles with these questions:

   - How are they the same?
   - How are they different?
   - How would you classify the triangles you found?
   - What are the names of all triangles formed by the diagonals of the hexagon?
   - Is there a pattern in the numbers of congruent triangles?

4. Repeat step 3 above to find convex and concave quadrilaterals and pentagons.

Lesson 3

AS THE ART OF OTHER CULTURES IS EXAMINED, students discover and describe the use of geometric concepts. Students apply the geometric concepts as they design their own tiles.

Activities and suggestions for discussion and exploration

1. Explore the art of other cultures shown in **figure 1.** Examples can be found on the Internet and in books. An entire lesson or multiple lessons may be dedicated to researching the art, or the research could be cooperatively integrated into another content area. Allow students to describe and discuss examples noting the similarities and differences––the characteristics that cultures share as well as those that make each unique.

2. As described in lesson 1, construct any regular polygon. Draw all possible diagonals within the polygon. Create additional line segments by connecting vertices that were formed by the intersection of diagonals. Circles and arcs can be added by using vertices (intersections on and within the polygon) as centers of the circles.

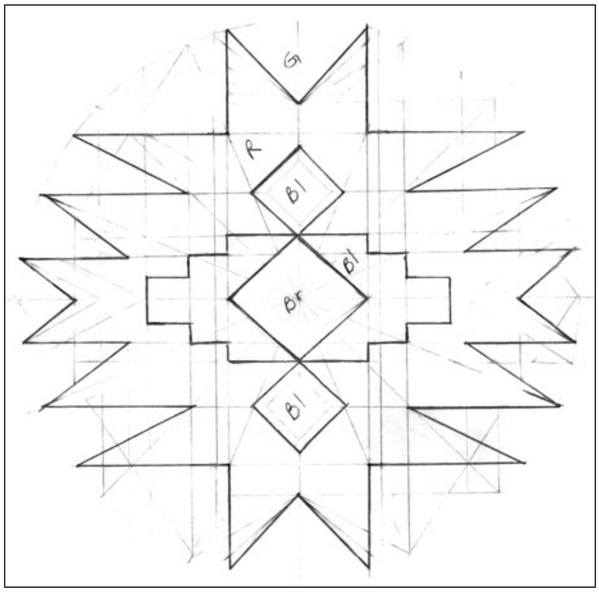


Fig. 6 Lines are erased to reveal the design.

3. Explore the possibilities! Visualize positive and negative space. Find shapes within the larger polygon (possibly by erasing line segments) to create a design for the tile, as illustrated in **figure 6.** Use colored pencils to bring out the designs and patterns and to plan a color scheme.

Lesson 4

STUDENTS RECREATE THE DESIGNS FROM LESson 3 onto bisque tiles and begin to underglaze. An application of underglaze will take a minimum of two class periods if performed with care.

The Final Phase: Working with the Tile

1. RECREATE THE DESIGN FROM LESSON 3 ON A bisque tile. If the tile is beveled, draw the circle just inside the beveled edge. Keep pencil marks light. Even though pencil markings are burned off in a kiln, heavy pencil markings sometimes prevent the underglaze from adhering to the tile. This particular characteristic of pencil lead on tile can be used to artistic advantage if white lines are desired as part of the design. *Cautionary note:* Designs can become extremely complex, resulting in very small spaces. Controlling the underglazing of very small areas requires skill and patience. The more complex the design, the more time and patience are required.

2. The teacher may need to demonstrate applying underglaze if the students do not have experience with this process. This task could be done in cooperation with the school art teacher. (If you are on your own, consult the store from

TABLE 1

Recording Sheet for Lesson 1

Regular Polygon (Inscribed in a Circle)	Number of Sides	Length of Arcs of the Circle (360º ÷ No. of Sides)	Number of Triangles (Created by the Diagonals Drawn from One Vertex)	Measurement of the Interior Angles of the polygon
	3			Total: ____ Each:
	4			Total: ____ Each:
	5			Total: ____ Each:
	6			Total: ____ Each:
	8			Total: ____ Each:
	9			Total: ____ Each:
	10			Total: ____ Each:
	12			Total: ____ Each:

which you purchased the product.) Once the tiles are completely underglazed, they should be allowed to dry, then handled carefully before firing.

Firing, glazing, and firing again

Tiles with underglaze go through one firing. To attain a shiny finish that seals the tile, apply two thin even coats of clear glaze to the tiles and fire again. When the final firing is done and the kiln is cooled down, open the kiln to experience, in the words of our art teacher, "the thrill of Christmas morning."

Options and Extensions

1. THE DESIGNS CREATED BY STUDENTS DO NOT have to be made into tiles. They can be recreated on pieces of heavy paper or cardboard and finished with colored pencils or markers.

2. Connect this mathematics lesson to communication by asking students to write the story of the tile. Include the decisions that were made in the process, the properties of the geometric figures, and the mathematical relationships among geometric figures. Explain any symbolic significance.

3. Completed tiles can be displayed, turned into a permanent art installation for the school, or taken home and used for decoration or as coasters or hot mats. Each student can create two tiles—one for a permanent installation at school and one for home.

Conclusion

THE GEOMETRIC TILE DESIGN PROJECT FOSTERS student engagement. The elements of choice and creativity inherent in the lessons allow them to be both authentic to the culture being studied as well as meaningful to students. Those students who tend to lag behind in other activities are more focused as they work to create their tiles. Students show gains in their facility using measurement tools and tools for construction. Most important, attention to properties of the figures is fundamental to the lesson as students construct all geometric figures for analysis and design.

Bibliography

Godlas, Sylvia. *Doorways to Islamic Art*. Arab World and Islamic Resources (AWAIR), 1997.

Hardaker, Chris. "Native American Geometry." www.earthmeasure.com. 1994–2000.

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics*. Reston, Va.: NCTM, 2000.

Web Sites for Viewing Native American Artwork

www.medicinemangallery.com
www.american-indian-art.com/baskets.html
www.puebloarts.com

Web Sites for Viewing Islamic Artwork

www.lacma.org/islamic_art/eia.htm
www.superluminal.com/cookbook/index_gallery.html

Web Sites for Viewing Mexican Artwork

www.homesteadmuseum.org/tile/
www.mexicanfolkart.com/tal_tile.html □