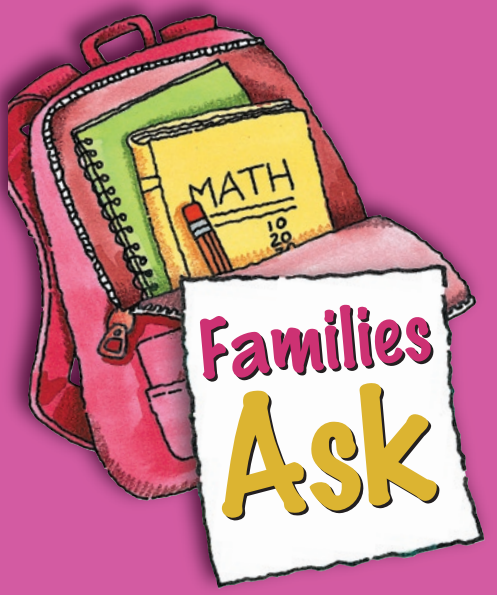

DAVID S. ALLEN

"Families Ask" helps classroom teachers respond to questions commonly asked by caregivers of their students. A commonly asked question will be posed; a rationale will be presented for teachers; and a reproducible page will be offered for duplication and distribution to anyone involved in the mathematics education of middle school students.

Here is this month's question:

I struggled with geometry in high school. How can I be sure that my child will have more opportunities for success than I did?

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Geometry: More than Just Shapes

Think back to the geometry you experienced as an elementary school student. Now recall a problem from high school geometry. Often, geometry tasks at the younger grades are limited to identifying shapes or labeling properties; in high school, students are expected to use abstract reasoning to prove a complex relationship. Instruction in geometry has traditionally been overlooked during middle school, which causes a gap between elementary school experiences and the thought processes required in high school. Highlighting this gap, Van Hiele (1999) identified five levels of thinking through which children progress: Level 0: Visual; Level 1: Descriptive; Level 2: Informal deduction; Level 3: Formal deduction; and Level 4: Rigor.

Lower-level experiences generally relate to knowledge development; the higher levels require more analysis and synthesis of ideas. According to Van Hiele, a major issue at the high school level is the lack of sufficient developmental experiences from middle school. It is quite a leap to move from elementary to high school concepts. The Geometry Standard defines geometry across the grade bands in a way that emphasizes the development of students' thinking about geometry (NCTM 2000). "Geometry enables us to describe, analyze, and understand our physical world, so there is little wonder that it holds a central place in mathematics or that it should be a focus throughout the school mathematics curriculum" (NCTM 2002).

It is critical for the middle school teacher to understand students' levels of geometric thinking. According to van Hiele (1999), middle-level geometry instruction should include geometric activities and manipulations so that students build an understanding of relationships. Middle school experiences should provide opportunities for students to order properties and formulate definitions based on explorations with shape. For example, as students explore the properties of a rectangle, they learn about angles and sides (e.g., four right angles, angle sum is 360 degrees, parallel sides), which form a foundation for identifying relationships that exist among other quadrilaterals. Students should be able to conclude, for example, that if all angles are right angles, the shape must be a rectangle. "If-then" statements provide a structure that students can use to develop an understanding of shapes and that also allow them to develop a network of relationships that will set the stage for a more advanced study of geometry.

As middle schoolers study geometry, their experiences should connect one concept of geometry to another. Students build on previous knowledge when they investigate area and perimeter of compound regions by dividing the complex region into simpler, more familiar shapes to which they apply familiar formulas. While studying transformations, students can examine concepts of symmetry and be asked, "Is there a relationship between the number of lines of symmetry within a specific shape and the angles of rotational symmetry of that same shape?" Students should experience problems that further develop concepts of location and spatial relationships, such as examining two-dimensional shapes on the coordinate system to connect properties of shapes and algebra, analyzing the slope of the sides of a triangle as they are transformed through manipulation, and exploring the change in the slope of a line as one side of the triangle is lengthened. Discussing explorations encourages students to make conjectures and focus on "if-then" statements. Teachers at the middle level are encouraged to use manipulatives to make geometry real for students while giving them the opportunity to think about geometry at progressively higher levels.

References

National Council of Teachers of Mathematics (NCTM). *Principles and Standards for School Mathematics.* Reston, VA: NCTM, 2000.

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Van Hiele, Pierre M. "Developing Geometric Thinking through Activities That Begin with Play." *Teaching Children Mathematics* 6 (February 1999): 310–16. □

Families Ask Take-Home Page

Families often ask a question like this:

I struggled with geometry in high school. How can I be sure that my child will have more opportunities for success than I did?

Have you ever noticed that when adults reflect on their experience in high school geometry they almost always either loved it or hated it? A person's geometry experiences in kindergarten through eighth grade may be the basis for that strong feeling in high school. Learning geometry takes much longer than a year; in fact, to be prepared for geometry in high school, students should explore it as early as kindergarten, and experiences should increase in difficulty throughout middle school. Geometry needs to be introduced in a meaningful manner so that students, over time, have opportunities to develop more complex thinking.

What can this study look like in grades 6–8? For example, when exploring shapes and their properties, students can look at the two figures below and see similarities (i.e., they both have four right angles; they both have opposite sides that are equal and parallel).

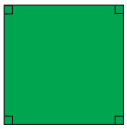


Figure A

Figure B

In middle schools, students should engage in activities that encourage them to look at the properties of the shapes (angles, sides, and so on) and begin to develop understandings about relationships among the properties. For example, middle school students should be able to make the following if-then statements:

- If all four angles are right angles, then the shape must be a rectangle.
- If it is a square, then all angles are right angles.
- If it is a square, then it must be a rectangle.

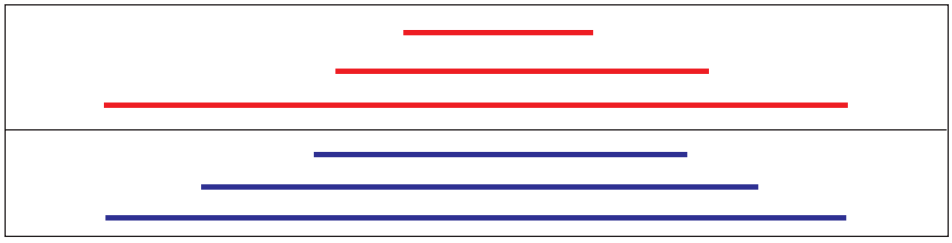
If-then thinking is one goal of middle-level geometry instruction. It is referred to as *informal deductive thinking* and is one step toward bringing students to a level where they can be successful working with proofs required by many high school geometry textbooks. To explore this form of thinking, try the following activity (Lappan et al. 2002).

Activity: Break pieces of uncooked spaghetti to match the lengths of the two groups of line segments shown in the figure at the bottom of the page. Then answer these questions:

1. How many triangles can you form from each group of spaghetti?
2. How can you tell by looking at the lengths of spaghetti whether or not they can be used to form a triangle?
3. Write an if-then statement that expresses your answer to question 2.
4. Create several more groups of spaghetti and test your statement in question 3.

You should have been able to determine that the two smaller lengths of spaghetti, when placed end to end, must be longer than the longest length of spaghetti. A teacher would look for students to make this statement, "If the sum of the two smaller lengths is not greater than the length of the larger piece, then the three pieces cannot be used to create a triangle." This is called the *triangle inequality property*. While doing this activity, you had a chance to manipulate objects, which is the same type of manipulation that teachers ask their students to do. The purpose of this activity and those activities that your child experiences in school is to develop an understanding of geometry by looking for relationships.

To help your child develop the knowledge and thought processes to be successful in geometry, teachers need to present students with a middle school curriculum that challenges their thinking about new relationships. This type of thinking not only prepares students for later study in geometry but connects with other areas of mathematics, such as algebra. You can contribute to your child's growth in geometry by talking with him or her about various activities. If students can communicate their thinking about geometry at home, their understanding will be strengthened. This is a great opportunity for you and your child to talk about how they are thinking about mathematics.



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Lappan, Glenda, James T. Fey, William Fitzgerald, Susan N. Friel, and Elizabeth D. Phillips. "Shapes and Designs: Two Dimensional Geometry." In *Connected Mathematics Project*, 19–21. Palo Alto, CA: Dale Seymour Publications, 1996. □

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