

Helping English-Language Learners Develop Computational Fluency

Students who are computationally fluent can solve problems accurately, efficiently, and with flexibility. These students draw on a repertoire of strategies when solving problems, and their choice of strategies often depends on the type of problem they are solving and the numbers involved.

Computational fluency is rooted in an understanding of arithmetic operations, the base-ten number system, and number relationships. Communicating mathematical ideas is fundamental to developing computational fluency. When students share their solution strategies with others, they learn that there are many ways to solve problems and that some strategies are more efficient than others.

Although we can define computational fluency and explain how it can be nurtured, the challenge is to ensure that all students attain fluency. How can we help all students, especially English-language learners, develop computational fluency if they have experienced mathematics as quiet, solitary practice of standard procedures? How do we make communication the focus of mathematics class so that mathematical conversations are productive and accessible to everyone? What sensitivity, awareness, and skills do teachers need when working with students from diverse backgrounds with differing experiences and skills who may be learning English as a second language?

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Communication: An Instructional Feature That Can Promote Fluency

Research in mathematics education identifies specific instructional features that promote conceptual understanding of mathematics and are associated

with higher levels of performance. One such feature is communication. Many researchers of mathematics learning have found that students benefit from communicating their mathematical ideas (Cobb et al. 1997; Heibert and Wearne 1993; Khisty 1995; Lampert 1990; Wood 1999). When teachers ask effective questions, they prompt students to articulate their various solution strategies, which can create a cross-pollination of ideas. Students become flexible problem solvers, and through shared dialogue they begin to build computational fluency. But what happens when students learning English as a second language are expected to communicate mathematical ideas in English?

Communication in mathematics class has the potential to facilitate understanding and develop computational fluency, but the practice of discussing ideas in English may place children who are learning English as a second language at a distinct disadvantage. For example, English-language learners can become confused during a discussion if the mathematics vocabulary has different meanings in everyday usage, as with *column*, *table*, and *rational*. They also may be confused if the same mathematical operation can be signaled with a variety of mathematics terms, such as *add*, *and*, *plus*, *sum*, and *combine*. A word such as *left*—as in “How many are left?”—can be confusing when the directional meaning of the word is most commonly used

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in everyday English. The words *sum* and *whole* also can cause confusion because they have nonmathematical homonyms. Furthermore, a symbolic statement such as $9 - 4 = 5$ can be expressed verbally in several different ways, such as “Nine take away four is five” or “Four from nine leaves five.” Unless teachers thoughtfully construct conversations that are intended to promote an understanding of mathematics, English-language learners are less likely to benefit from mathematical discussions.

If students talk about their mathematical ideas in order to develop their computational fluency, teachers must make sure that communication does not result in inequity for English-language learners. The Equity Principle in *Principles and Standards for School Mathematics* (NCTM 2000) states that all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study and learn mathematics. NCTM recognizes that equity requires an accommodation of differences. Teachers must pay special attention to classroom discussions so that students who are not native speakers of English can equally participate. Secada (1996) suggests that mathematics teachers use what bilingual educators have learned to help English-language learners communicate their mathematical ideas in English, thereby helping them develop computational fluency.

Strategies for Helping English Learners

Computational fluency requires the kind of flexibility in thinking that enables a student to choose an

efficient strategy for a particular problem to yield a correct answer. In order for students to become flexible problem solvers, they must be aware that numerous strategies for finding an answer may exist. By participating in mathematical discussions, students can become aware of a variety of strategies. If these discussions involve English-language learners, teachers must provide extra support.

I based the strategies in **figure 1** on my own teaching experiences and the ideas of bilingual educators (Diaz-Rico and Weed 1995; Garrison 1997; Khisty 1995; Secada 1996). These suggestions have two goals: to help English learners communicate their mathematical ideas in English and to support their thinking of different strategies for solving problems.

Classroom Discussions That Develop Computational Fluency

While spending time with a class of second and third graders during their mathematics class last year, I was reminded of how difficult it can be for students to communicate their mathematical ideas and to think of different strategies for solving problems. Trying out my discussion strategies helped me assist the students, all of whom were English-language learners with varying levels of English language competency. Some students were experiencing instruction in English for the first time, while others were fairly skilled but not quite fluent. Spanish was the native language of all the students.

Ten strategies for helping English-language learners

1. Ask questions and use prompts.

- What do you think the answer will be? Why do you think that?
- What did you do first? Second? What do you need to do next?
- You figured it out by . . .
- You said that you counted. How did you count?
- What is the problem about? Do you think you will add or subtract?
- Is there another way to solve that problem? How about starting in the tens place instead of the ones?
- Is that number close to a friendly number you know?
- Can you break apart that number in some way?

2. Practice wait time. After asking a question, wait for a while before calling on a volunteer. This gives English-language learners time to process questions and formulate responses.

3. Modify teacher talk. Speak slowly and use clear articulation. Reduce the amount of teacher talk, use a variety of words for the same idea, exaggerate intonation, and place more stress on important new concepts or questions. Model or gesture when possible to supplement the verbal discussion.

4. Recast mathematical ideas and terms. Mathematics has many linguistic features that can be problematic for English-language learners. Use synonyms for mathematical words, such as *subtract*, *take away*, and *minus*. At the same time, be aware that using too many terms simultaneously can confuse the English-language learner.

5. Pose problems that have familiar contexts. When a problem is embedded in a familiar context, English-language learners have an easier time understanding the problem's structure and discussing how to solve it.

6. Connect symbols with words. When strategies for solving problems are described, write the number sentences and point to the symbols (such as $+$, \times , $=$), stressing the words in English.

7. Reduce the stress level in the room. Create a low-stress environment that encourages expression of ideas, where mathematical mistakes are seen as opportunities for learning and linguistic mistakes such as incorrect grammar do not inhibit the recognition of good mathematical thinking.

8. Use "Think-Pair-Shares." In this activity, students think about an idea, share the idea with a partner, then share the idea with the class.

9. Use "English experts." A student explains a strategy in her native language to a more capable English speaker, then the "English expert" translates the strategy for the teacher.

10. Encourage students to "retell." This is when a student is asked to explain a strategy, in English, that someone else in the group might have used.

The following discussion occurred while four students were playing "The Game of Pig" (Burns 2000). As I watched them play, I used several different strategies to help students carry on a productive mathematical conversation. In this game, students roll two number cubes and add the numbers to find the sum. The goal is to be the first to reach one hundred. If a player rolls a one, he loses all the points earned in a round. If he rolls double ones, he loses all his points and must return to zero. Games of this type engage and motivate English-language learners and provide a familiar context for dis-

cussing arithmetic strategies. In the following, Noel acts as the "English expert" and Lenny retells Iziquiel's "doubles minus one" strategy.

Noel. Seventeen plus 10 equal 27.

Teacher. How did you get 27?

Noel. I add 10 plus 10; that makes 20, and 20 plus 7 is equal to 27.

Teacher. Noel, will you tell Anton in Spanish how you solved the problem?

Noel. Diez mas diez igual a veinte. Veinte mas siete igual a veinte y siete. [*Ten plus 10 equals 20.*]

Twenty plus 7 equals 27.]

Teacher: Anton, do you understand?

Anton: Yes. [rolls the number cube] Six plus 3 equals 9.

Teacher: How did you get 9? [Anton does not respond.] Noel, will you ask Anton to tell you how he got the answer to 6 plus 3, then tell me in English?

Noel: ¿Como supiste que seis mas tres es nueve? [How did you know that 6 plus 3 is 9?]

Anton: Empeze a contra del seis. [I counted on from 6.]

Noel: Anton counted on from 6.

Iziquiel: [rolls a 5 and then a 4] Five plus 5 is 10, and 10 minus 1 is 9.

Teacher: Iziquiel, will you explain your strategy to us?

Iziquiel: I know 5 and 5 is 10. But 5 plus 4 is like, 4 is 1 less than 5, so 5 plus 4 is 9.

Teacher: Lenny, please retell how Iziquiel solved the problem in English.

Lenny: [pause] He did 5 and 5 is 10, then he takes away 1 to make it 9.

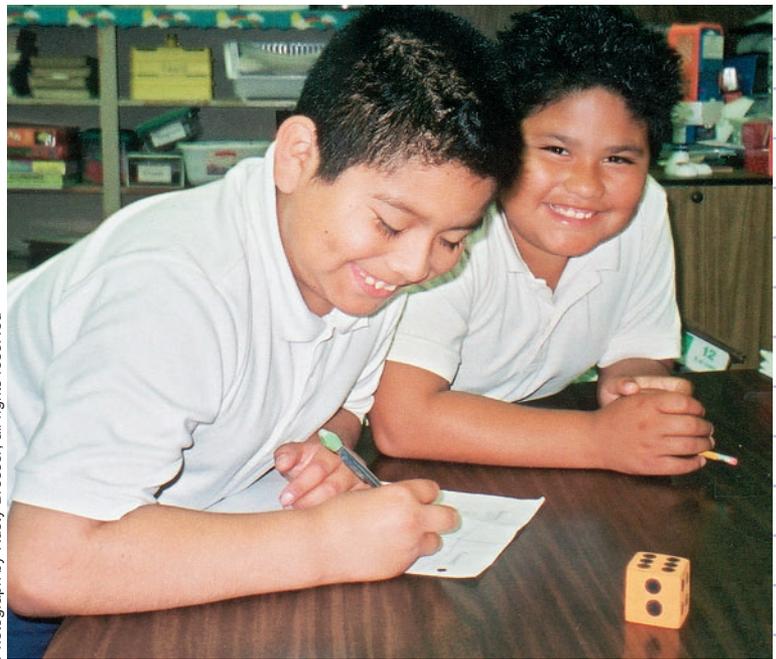
Having a group discussion during the game allowed several strategies for combining numbers to surface. Iziquiel's "doubles minus one" strategy reflects a certain level of computational fluency: He uses his knowledge of number relationships to solve the problem. Having "English experts" provide translations gives beginning English speakers like Anton access to new ideas. And asking students like Lenny, with more highly developed English language skills, to "retell" someone else's strategy helps those participating in the conversation understand more efficient methods.

The next discussion took place after I posed the following problem: "Maria had 49 playing cards. Antonio gave her some cards. Now she has 84 cards. How many cards did Antonio give her?"

I asked the students to solve the problem mentally; whenever possible, I also provide models and manipulatives. Unlike the small-group conversation during "The Game of Pig," the discussion that follows included the entire class. Although facilitating a discussion with a large group can be challenging, it exposes students who are less skilled in English, like Anton and Jesús, to a broader range of ideas.

As the students shared their strategies, I recorded their ideas on the chalkboard. I used the "Think-Pair-Share" activity to give students some initial practice with describing their strategy to a friend before beginning a whole-group discussion.

Teacher: Think about a way to solve the problem I just gave you. Then pair up with a partner and share your strategy. [After giving the class some



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time to think and share their thoughts, I called on Marla.]

Marla: I did 84 take away 49. I couldn't do 4 take away 9 so I borrowed a 10 from the 8 and gave it to the 4 to make 14. Then I did 14 take away 9 is 5 and 7 take away 4 is 3, so it's 35 cards Antonio gave Maria.

Teacher: Who solved it a different way?

Jesús: I counted up from 49 to 84 and I got the same answer as Marla.

Teacher: Who solved it in a different way?

Christina: I did 49 plus 13, but I didn't get 84 for an answer, so I tried 49 plus 30 and I got 79. That didn't work, so I tried different numbers till I got up to 84. I finally got 49 plus 35.

Teacher: So Christina used trial and error. She tried different numbers until she got the right answer. Another way?

Anton: I count back by ones.

Teacher: What number did you start with?

Anton: Eighty-four.

Teacher: What number did you stop at?

Anton: Forty-nine.

Teacher: And how many numbers did you count back?

Anton: I got lost.

Daniela: I started with 49 and I added 5 to get 54. Then I did 54 plus 10 and got 64. Then I did 64 plus 20 to get to 84. I added 5 plus 10 plus 20 and got 35.

Teacher: Any other ways? [no response] OK. I want you to look at the numbers in the problem. Are the numbers close to any friendly numbers you know of? If so, can this help you think of another way to solve the problem?



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Daviel. You could make the numbers 84 and 50. Fifty is close to 49. So 84 take away 50 is 34. But I have to add one on 34 'cause I added one to 49. That makes the answer 35.

Daniela. I know! You could do 85 minus 50. If you add one to 84 you can add one to 49 to make it easy. The answer is 35.

In this discussion, several of the students' strategies were inefficient or prone to error. Although Marla arrived at the correct answer by using the standard algorithm, children who use this method often make mistakes because keeping track of the regrouping is difficult, especially when solving problems mentally. Jesús and Anton's strategy of counting by ones also was inefficient and prone to error, especially because 84 is much greater than 49. Although creative, Christina's trial-and-error strategy also was inefficient. Daniela's use of making leaps of five, ten, and twenty to solve the problem was the most efficient strategy and revealed her working knowledge of base-ten number concepts.

After it seemed that the students had exhausted all their ideas, I tried to help them consider more efficient strategies by prompting them to use friendlier numbers that are easier to work with. This hint led Daviel to use a strategy that Carpenter et al. (1999) call "compensating." First, he added 1 to 49 to make the friendlier 50. Then he subtracted 50 from 84 and got 34. Finally, he added 1 to 34 to compensate for the 1 that he initially added to 49. Daniela used a similar strategy that Fosnot and Dolk (2001) describe as "constant dif-

ference": She added the same amount—1—to both numbers, then subtracted. Both Daviel's and Daniela's strategies reflect computational fluency: They are efficient and appropriate for the problem type and the numbers involved. Furthermore, both methods often yield correct answers when students understand the ideas. If Daviel and Daniela had not come up with their ideas, I might have modeled these methods for the class and posed other problems for which they are appropriate.

Teachers can facilitate mathematical discussions such as the one above through activities like "Think-Pair-Shares," by asking questions that prompt student reflection such as "What number did you start with? How many numbers did you count back?" and by giving children access to one another's ideas. Although these teaching strategies are likely to be effective with all students, they are especially important for English learners, who many need extra support when engaging in oral language activities in English.

Making Communication a Focus During Mathematics Class

Garrison (1997) points out that many educators share the misconception that because it uses symbols, mathematics is not associated with any language or culture and is ideal for facilitating the transition of recent immigrant students into English instruction. If teachers view mathematics as language-free, they likely will breathe a sigh of relief when mathematics time comes and think, "Finally! A subject where my second-language learners do not have to struggle with English!" Consequently, students often end up working on "word-free" worksheets, practicing only standard procedures for solving problems, and experiencing mathematics as something that is done quietly in isolation.

The students I worked with last year had indeed experienced mathematics as the drill and practice of standard algorithms, which they practiced alone by doing worksheets. When I initially asked them to solve problems in ways other than by using standard procedures, many of these English-language learners reverted to inefficient methods such as counting. They did not have a repertoire of strategies from which to choose because they had not been exposed to the idea that many ways to solve problems exist. They also were not accustomed to discussing their mathematical ideas in English.

As soon as communication became the focus of mathematics class, however, students began to make progress in their mathematical thinking. This process did not come easily; it took time and did

not happen merely because students were allowed to talk. My role as the teacher was to create a safe environment for expressing ideas, model mathematical talk, provide mathematics games for students to work on in small groups that encourage conversations, and moderate discussions to make sure that the talk was productive and focused on the mathematics. Over time, the class became less resistant to my expectation that an explanation must accompany an answer to a mathematics problem.

When facilitating productive talk during mathematics class, teachers can help ensure that emergent English speakers fully participate by structuring discussions in ways that provide access to students with varying linguistic expertise. By using prompts, asking questions, and encouraging mathematics conversations, teachers accomplish two valuable goals: English-language development and computational fluency.

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