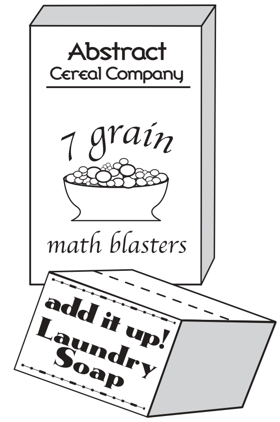
Packing the Packages



Did you ever notice that cereal comes in tall, thin boxes and that laundry soap comes in short, wide boxes? Is the way a product is packaged important? What box shape holds the most and uses the least amount of material to make? Let's explore the amount of packaging material needed to wrap a product.

1. Most cereal boxes are right rectangular prisms. That is, they have two parallel rectangular regions called *bases,* which are connected by four other rectangular regions called *lateral surfaces*. Suppose that your favorite cereal comes in a box that is 12 inches high, 10 inches long, and 3 inches wide. Sketch a picture of the box on a separate sheet of paper, and label it with the dimensions.

2. *a*) What are the dimensions of the bottom of the box? ________________

   b) What are the dimensions of the front of the box? ________________

   c) What are the dimensions of the side of the box? ________________

3. What part of the cereal box has the same dimensions as the bottom panel?

   ____

4. Finding the amount of cardboard needed to make the cereal box is sometimes easier if you use a flat pattern of the box. This flat pattern is called a *net*. On a separate sheet of paper, draw a sketch showing the cereal box if you cut it apart and flattened it.

Take an empty cereal box, and cut along as many edges as necessary to lay the box completely flat and keep it in one piece. Did your sketch match the box?

To calculate the amount of material needed to make the box, you must find the surface area of the box.

5. How many rectangular regions make up the net? ________________

6. On your sketch, label the front, back, top, bottom, and the right and left sides of the box. Also label the dimensions of the box.

7. *a*) Find the area of the front panel of the cereal box.

   ____

   b) Find the area of the top panel of the cereal box.

   ____

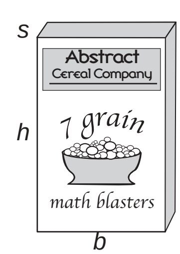
   c) Find the area of one of the side panels of the cereal box.

   ____

8. Find the total surface area of the box.

   ____

9. You can develop a formula for the surface area, *SA,* of all boxes that are rectangular prisms by representing the different edges of a box with variables. Assume that a box is sitting on its bottom surface with its front panel facing you, as pictured at right. Let b represent the bottom front edge of the box. Let s represent the top edge on the left side of the box. Finally, let h represent the left edge of the front panel of the box.



Write an equation that represents the surface area (*SA*) of the box.

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The editors wish to thank Sheryl Yamada, Beverly Hills High School, Beverly Hills, CA 90212, for the ideas used in this issue of *Student Math Notes.*

10. The amount of space inside the box is called its *volume*. You can find the volume of the box by calculating the number of one-unit cubes needed to fill the box.

    a) If you begin to fill the box with one-inch cubes, how many one-inch cubes would complete the first layer in the bottom of the cereal box? ______________________

    b) How many layers of one-inch cubes are needed to fill the whole box? ______________________

    c) Find the number of one-inch cubes needed to fill the box. ______________________

    d) Write an equation that relates the dimensions of the box to the total number of one-inch cubes that would fill the cereal box. ______________________

11. Using the same variables that you used in problem 9, write an equation that represents the volume, *V,* of the cereal box. ______________________

12. Notice that the number of one-inch cubes in the bottom layer of the box is the same as the number of square inches in the area of the bottom panel. Will you always obtain this result? Explain your reasoning.

M.A.T.H. Corporation manufactures manipulatives for mathematics classrooms. One popular manipulative is a set of 100 blocks that are one-inch cubes. M.A.T.H. Corporation is trying to find ways to decrease its packaging costs. The company has decided to arrange each set of blocks in a box that is shaped like a rectangular prism.

13. Complete the chart to find all the possible box arrangements for 100 blocks. Turning a box or standing it upright does not constitute a new or different box shape. Find the surface area and the cost to manufacture the box if boxes can be made for 0.8¢, or $0.008, per square inch. Since the box must hold exactly 100 one-inch blocks, the volume of each box will be the same.

Base Front	Side	Height	Volume	Surface Area (sq. in.)	Cost ($0.008 per sq. in.)
100	1	1	100		
50			100	304	
			100		
			100		
			100		
	10	1	100		$1.92
			100		
			100		

14. *a*) What are the dimensions of the box that costs the most to manufacture? ______________________

    b) What does this box look like? ______________________

    c) Explain why this shape is the most expensive one for the box to have.

15. *a*) What are the dimensions of the box that costs the least to manufacture? ______________________

    b) How does its shape differ from that of the most expensive box?

    c) Why would you expect this box to cost less?

    d) If the box did not have to hold the cubes but still had a volume of 100 cubic inches, what would be the shape of the least expensive box? ______________________

16. An employee suggested that 125 one-inch blocks could be packaged for the same cost as 100 blocks.

    a) What are the dimensions of the box that would most economically hold 125 one-inch blocks? ________________

    b) Find the surface area and the cost of this package. ________________________________

    c) Is the employee correct? Explain.

M.A.T.H. Corporation wants to ship the manipulatives in cartons that hold exactly 100 packages of the blocks. Using the 5-inch-by-5-inch-by-4-inch box from the chart in problem 13, design at least two cartons that hold 100 packages of the blocks.

17. *a*) On a separate sheet of paper, sketch your designs, and describe how the packages must be arranged to fill your cartons.

    b) The company wants the shipping cartons to be as compact as possible to keep costs to a minimum. The carton with the least amount of surface area would be the cheapest. Describe the shape that this carton must have.

    c) What are the dimensions of the carton with the minimum surface area that would hold 100 packages of the one-inch block manipulatives? ________________________________

M.A.T.H. Corporation also sells balls as manipulatives. The balls come in sets of three and are 3 inches in diameter. The marketing team is trying to decide whether a rectangular prism is still the best design or whether a cylinder would require less packaging material.

18. *a*) On a separate sheet of paper, sketch possible configurations in which three balls could be packaged.

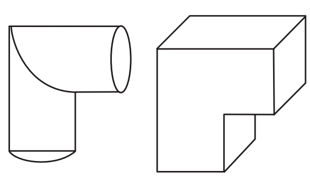
    b) On a separate sheet of paper, sketch possible packages to hold each of the arrangements of the balls. Label the dimensions of each package.

    c) Determine the surface area of each package.

    d) Which package would you recommend to M.A.T.H. Corporation? Justify your recommendation.

Can you . . .

- determine the surface area of these containers designed to hold three balls with 3-inch diameters?



- find the surface area and volume of a sphere?
- determine the relationship of the volume of a cylinder, cone, and sphere when they all have the same diameter and when the height of the cone and the cylinder equal the diameter of the sphere?

Did you know . . .

- that a sphere always has a smaller surface area than a cube with the same volume?

Mathematical content

Surface area, volume, optimization, spatial visualization

References

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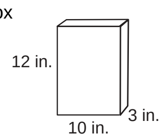
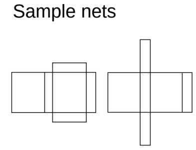
Johnson, Christine V., and Nancy Cook. *Designing Environments*: *Real-World Mathematics through Science*. Orangeburg, N.Y.: Dale Seymour Publications, 1998.

Answers

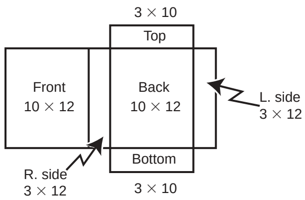
1. Sample box



2. (*a*) 10 in. × 3 in.; (*b*) 10 in. × 12 in.; (*c*) 3 in. × 12 in.
3. The top panel
4. Sample nets



5. Six rectangular regions
6. Sample net

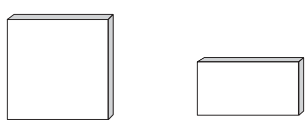


7. (*a*) 120 sq. in.; (*b*) 30 sq. in.; (*c*) 36 sq. in.
8. 372 sq. in.
9. $SA = 2bs + 2hs + 2bh$.
10. (*a*) 30 cubes; (*b*) 12 layers; (*c*) 360 cubes; (*d*) Total cubes = $3 \times 10 \times 12$.
11. $V = bsh$.
12. Yes; the faces of the cubes that are touching the bottom of the box exactly cover the same area as the bottom panel.

13. Sample chart

Base Front	Side	Height	Volume	Surface Area (sq. in.)	Cost ($0.008 per sq. in.)
100	1	1	100	402	$3.22
50	2	1	100	304	$2.43
25	2	2	100	208	$1.66
25	4	1	100	258	$2.06
20	5	1	100	250	$2.00
10	10	1	100	240	$1.92
10	5	2	100	160	$1.28
5	5	4	100	130	$1.04

14. (*a*) 100 in. × 1 in. × 1 in.; (*b*) The box would be long and skinny; (*c*) It has the greatest surface area.
15. (*a*) 5 in. × 5 in. × 4 in.; (*b*) It is more compact, and its shape is closer to that of a cube; (*c*) It has less surface area; (*d*) A cube.
16. (*a*) 5 in. × 5 in. × 5 in.; (*b*) The box would require 150 square inches of material and would cost $1.20 to produce; (*c*) No; 100 cubes can be packaged for less, since the 5 in. × 5 in. × 4 in. box has a surface area of 130 square inches and costs $1.04.
17. (*a*) Sample cartons

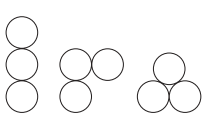


One layer of packages 10 down and 10 across

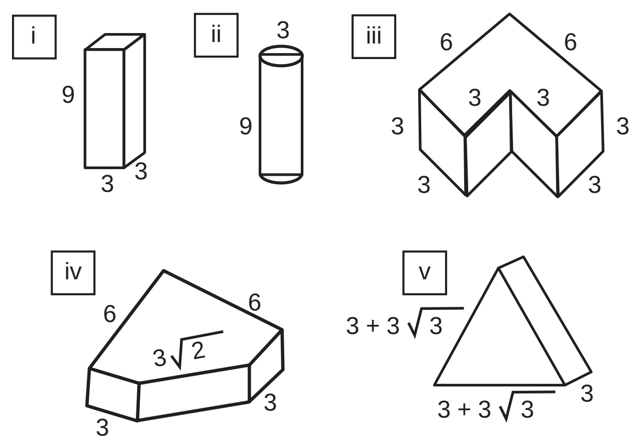
Two layers of packages 5 down and 10 across

(*b*) It must be close to a cube; (*c*) 25 in. × 20 in. × 20 in.

18. (*a*) Possible arrangements



(*b*) Sample containers



(*c*) The surface area of the rectangular prism (fig. i), cylinder (fig. ii), and triangular prism (fig. v) are 126 square inches, approximately 99 square inches, and approximately 132 square inches, respectively. Figure iii has a surface area of 126 square inches. Figure iv has a surface area of approximately 130 square inches; (*d*) The cylinder should be recommended, because it has the least amount of surface area and would cost the least to produce.

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