Developing Spatial Understanding through Building Polyhedrons

What kind of spatial understanding do first and second graders have? What do they see when they look at three-dimensional objects? What words do they use to describe what they see? How might their visualization skills be sharpened by building and describing three-dimensional structures? These questions emerged as we, a teacher educator and a classroom teacher, considered infusing spatial tasks into the primary school mathematics curriculum.

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We were concerned that our students were not given opportunities to develop their spatial abilities. Developing spatial thinking has been neglected in school environments to the point that students who have exceptional ability in this domain reported disenchantment with school and were far less likely to pursue advanced academic degrees than their peers who have comparable abilities in other domains (Gohm, Humphreys, and Yao 1998).

Spatial intelligence, identified by Gardner (1983) as one of the seven intelligences, involves "picturing things in the head" and includes the abilities to imagine folding and turning an object, to visualize the three-dimensional object associated with a two-dimensional drawing, and to recognize pictures of the same object drawn from different perspectives. Boys often perform better on tests of spatial intelligence than girls do. Researchers speculate that boys' performance may result from their play preferences, such as building with blocks, negotiating computer environments, and so on, which give them more opportunities to practice this kind of thinking. NCTM's *Principles and Standards for School Mathematics* (2000) points out the importance of giving children the opportunity to develop visualization and spatial reasoning through work with concrete objects. Our interest in gender issues and in improving mathematics instruction led us to experiment with incorporating spatial tasks into the early elementary school curriculum. We chose to use Polydrons, plastic triangle, square, pentagon, and hexagon shapes that snap together easily, to create two- or three-dimensional objects.

We worked in a combined first- and second-grade public school classroom in which the children entering first grade stay with the teacher until they complete second grade. During the two-year period in which we explored our questions, the class had about twenty children, about the same number of boys and girls and the same number of first and second graders. While we explored spatial thinking with the children, we took notes about their work; these notes provided the data for the vignettes used in this article. All student names are pseudonyms.

Our overall plan for instruction was to engage

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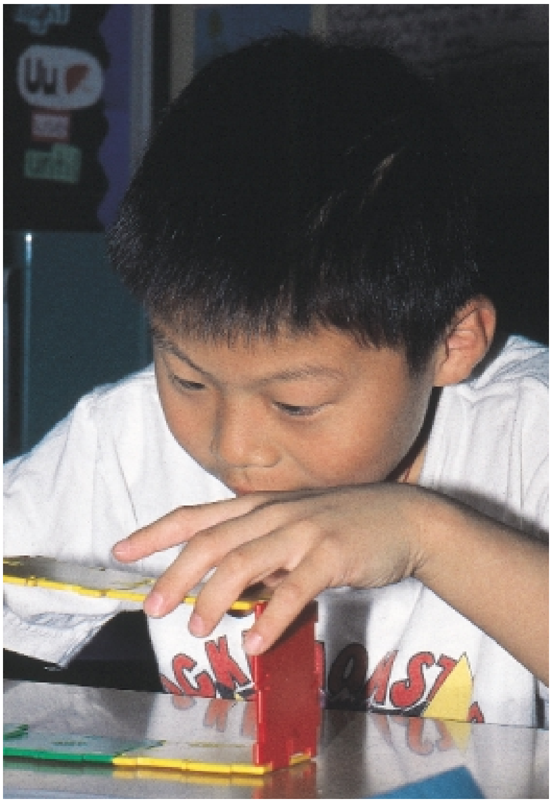


the children in building and describing polyhedrons. As they built polyhedrons, the children exercised their creativity, making unique and complicated structures. The children then described their creations. Our hope was that after several iterations of building and describing, the children would develop more detailed descriptions using more sophisticated language and begin to recognize some of the subtler attributes of polyhedrons. The creative and physical aspects of our lessons led to a high level of engagement for all the children. Each time the children had opportunities to build with the Polydrons, they built more intricate and complicated structures than they had before.

This article shares the results of our exploration into children's thinking. We reflect on the sequencing of the tasks in light of our findings. We also consider our questioning techniques to determine lines of inquiry that could extend children's thinking.

Closed versus Open Polyhedrons

We began with an introductory session in which the children built and described all kinds of structures; then we introduced the concept of closed structures. We gathered the whole class and asked the students to compare an open and a closed structure that two of the children had created. The children commented, "That one is a drum and this one is a car"; "That one is bigger"; "That one has more squares than that one"; "One could fit inside the other." We chose to highlight and extend this last observation by asking, "How can one fit into another?" One child observed that the bigger structure had a hole that the smaller structure could fit into.

At this point, we introduced the conventional terms *open* and *closed* and told the children that mathematicians used this vocabulary to talk about structures with and without openings. The children readily adopted these terms because this attribute of three-dimensional objects was obvious to them and because the words were familiar. This attribute was often the first one that children noticed about structures, and the children remembered it even when they had not worked with polyhedrons for months.

As we worked with the class, we developed an approach to geometric terminology that we believed was appropriate for young children and in keeping with recommendations in *Principles and Standards* (NCTM 2000). We believed that the children could learn a few new words, but we did not want to focus on technical terms that were not commonly used. For this reason, we introduced the term *vertex* but were satisfied when the children called vertices "corners." We sometimes referred to the structures that children made as *polyhedrons,* and we told the children what this word meant. We did not expect the children to adopt this term and were comfortable when they talked about their "closed structures." By the end of the year, we had introduced the terms *face, edge, vertex, prism, cube,* and *pyramid.* We decided that appropriate use of familiar words was a suitable goal for first and second graders.

Describing Polyhedrons

After the class established a definition for *closed* structures, we asked the children to build one (a polyhedron), then to write a description of it. Both girls and boys were eager to begin building their structures, demonstrating that the activity was engaging to both genders. As they built, some students found that they could not close their structures. They needed to engage in spatial reasoning to solve the problem of how to change the structure so that it could be closed. They often consulted with classmates or looked at a completed polyhedron to get ideas of how they could change theirs. We elicited and stimulated their thinking by asking, "What shape would you need to fill this space in?" Sometimes, the children recognized that the required shape was not in the Polydron set, and sometimes they realized that they could fill in the hole by putting together a few pieces.

The children made a variety of structures, some

FIGURE 1

Children engage in building closed structures.

relatively simple, consisting of five or six pieces, and some complex, with up to twenty pieces (see **fig. 1**). We were impressed with the degree of imagination that the children brought to their work. When the children were finished building their closed structures, we asked them to come to the rug to describe what they had made. The following dialogue took place:

Ms. Ambrose. Anna, you have an interesting structure. Can you describe it?

Anna. Well, it's a big house. It kind of looks like a motel.

Ms. Ambrose. What shapes did you use to make it?

Anna. Squares and triangles. I made squares out of two triangles.

Ms. Ambrose. Did somebody else build with triangles?

Laura. I used two squares and eight triangles. See? It rocks back and forward.

Seth. I made a tent.

Ms. Ambrose. What shapes did you use to make it?

Seth. I used three squares and two triangles.

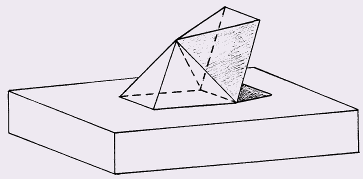
In reflecting on this discussion, we realized that it followed a pattern described by Lehrer, Jenkins, and Osana (1998). The children first described their creations in holistic terms, as houses, spaceships, tops, or other familiar objects. They had to be encouraged to identify the shapes that they used to make their structures. They were comfortable talking about triangles and squares but had trouble remembering the terms *pentagon* and *hexagon*. Many children discussed and demonstrated how their structures could be moved.

The Two-Square, Six-Triangle Problem

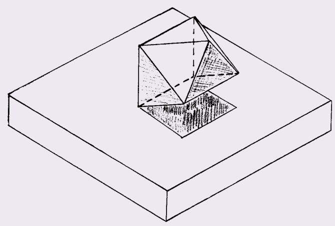
The next task grew out of our observations of the children's work on the previous task. We wanted to see whether the children could look at the relationships among the shapes, in addition to noting the shapes that they used. Could they observe, for example, "The square is surrounded by triangles"? Could they notice the edges of their structures and note which shapes shared an edge? We hypothesized that talking about edges would come naturally because when building with Polydrons, children have to snap faces together to make edges. The children's task was to build as many different structures as they could using six triangles and two squares. The task came from a structure that one of the first graders made (see **fig. 2a**). We believed that if one student could produce that polyhedron, then many of her classmates could do so, as well. We developed the task because we wanted children to see that enumerating the shapes in a polyhedron was

FIGURE 2

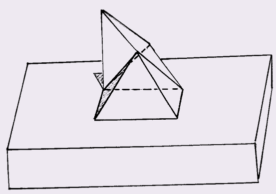
Structures built from two squares and six triangles



(a)
Adjacent squares



(b)
Adjacent squares



(c)
Nonadjacent squares

insufficient to adequately describe the structure. In this instance, three polyhedrons fit the description that “It has six triangles and two squares.”

We were surprised at how easily the children built their structures. In ten minutes, each child had built at least one polyhedron, and three different kinds had been produced for the class to examine (see **fig. 2**). In two of the structures, the squares shared an edge (**fig. 2a** and **2b**). In one of them, the squares were not adjacent (**fig. 2c**). When we chose the task, we hoped that the children would clearly see this difference.

We gathered the whole class to analyze the three different polyhedrons that had been formed and engaged in the following discussion:

Ms. Ambrose. Is your polyhedron different from your neighbor’s? If it is, how is it different?

Sarah. Mine’s an ice-cream cone, and his is a shoe.

Ger. I have a tent.

Tiffany. Mine is a car that can drive around. His is a top that can spin.

Shandor [who had all three structures]. These two are made out of pyramids, and this one is not.

Ms. Ambrose. Can you show us what you mean? [Shandor takes one of his structures apart and shows that it is made of two square pyramids, each missing a triangular face.]

Shandor. I made this last one by making two pyramids and sticking them together in a new way.

Ms. Ambrose. Can anyone else explain that? I’m not sure everyone understands.

Kaylah. See, if you take this [holding up her structure] and cut it here, you get two pyramids.

The discussion raised several issues. First, the children persisted in seeing their structures as being fanciful objects that could spin and drive around. We began to realize that given their age, this perspective was natural and showed their fascination with their creations. We realized that we should not expect them to abandon this orientation and, instead, should hope that they could entertain a geometric orientation alongside their fanciful one. Another time, building on this orientation might be profitable. We might have asked the children, “What makes that structure like an ice-cream cone?” to see if they talked about the point at the bottom. We could introduce vocabulary, such as *vertex,* to correspond with the features that they noticed.

The second issue raised by the discussion was that the children did not notice that the organization of the faces mattered. They recognized that their structures differed, but they did not talk about the fact that in two structures (**figs. 2a** and **2b**), the squares were next to each other, and in the other one, they were not. The Polydron tool may have contributed to the children’s lack of attention to the edges because in some constructions, the edges are not sharply defined. We were cautious about training the children to look at edges, however, because we did not want counting edges to be a rote exercise. We wanted the children to have a reason for attending to edges.

The third issue that arose was that some children dissected their structures into smaller three-dimensional parts. We had not expected this approach to analyzing three-dimensional objects. We were impressed that Shandor and Kaylah were familiar with pyramids and could see them as parts of their larger structures. They did not look at the structures only as whole objects; rather, they examined specific attributes. We had not anticipated that the children could look at the structures in this way and had not developed a line of questioning that would extend this perspective. We wondered whether this perspective would be more accessible to the children than examining edges.

Matching Descriptions with Structures

A third task assessed how children interpreted geometric descriptions and proved to be much easier than the others. The children were asked to match descriptions written by their classmates to their corresponding structures. We sat in a circle, putting the structures in the middle in full view and safely out of reach of eager hands.

Ms. Falkner [reading]. “Eight faces, 12 corners, 6 sides, 13 shapes, 18 edges.” Which structure does this describe?

Gretchen [reaching for a structure]. It’s this one. See? It has 1 hexagon, 6 squares for sides, and 6 triangles. That makes 13 shapes.

Ms. Falkner. Did Gretchen pick one that fits the description? [The class agrees.]

Kathy. Yes. That’s the one I made, and that’s my description.

Ms. Falkner. How can it have 13 shapes and only 8 faces? I don’t get it.

Kathy. See? I used 6 triangles for 1 face.

Ms. Falkner: OK. Here’s another one [reading]: “All triangles. It looks like a flying saucer with a triangle pyramid on top.”

Larry [reaching for a shape]. See. It has all triangles and it kinda looks [like] a flying saucer.

Seth. No. That isn’t it. It’s my description, but that isn’t what I built.

Larry. Well, you didn’t tell us how many triangles.

Ms. Falkner. Is there another structure that fits

Seth's description?

Ernie [reaching for another polyhedron]. See? This one is all triangles and it looks more like a flying saucer.

Seth. Yes. That's mine.

The discussion went on in this form and had the potential to help children see that a more complete description was required to single out one structure from all others. When Kathy wrote "6 sides" in her description, she was referring to the six square faces that were attached to the hexagonal base of her structure. Although *lateral face* would be the more appropriate term and *side* is usually reserved for two-dimensional figures, we chose not to pursue these distinctions at this point.

This task was relatively easy because incomplete information is often sufficient to match a description to its structure. If only one structure has eight faces, then the appearance of the edges is irrelevant. Using this task as a starting point, we could ask the children to further analyze the polyhedrons: "Which one has a square that shares an edge with a triangle?" This line of questioning would focus children's thoughts on the arrangement of the shapes in the polyhedrons. We might also have asked, "Which one is made of a pyramid and a cube?" This line of questioning would have built on the decomposition ideas that the children had begun to explore.

Reconstructing Structures

Another task involved having the children reconstruct a polyhedron from a description. We asked the children to build a structure, then to write a description of it. Even though the children had been given this assignment earlier, they were happy to do it again; indeed, most built something different than they had before. We saved these descriptions and gave them back to the students about ten days later. We asked the students to rebuild their original structures using their descriptions.

A few children referred to their original descriptions, but most assured us that they remembered what they had built and could re-create it. Their repeated building experiences seemed to have helped them develop spatial memory, the ability to store and recall three-dimensional structures. The fact that they did not use their descriptions also suggested that the descriptions were not very useful to them in re-building their polyhedrons. They knew which shapes to use, but their descriptions did not give them specific information about how to assemble the shapes. They still did not attend to the edges. When they had to decide how to put their shapes together, they continued to use a trial-and-error approach rather than a more analytical one. Because we failed to keep any of our own records of their original structures, we had to trust that the new polyhedrons the children built were, indeed, duplicates of the originals.

In the future, we would try this activity again but would manage it differently. Children could describe a polyhedron provided by the teacher to ensure that a record was kept of the structure in question. The description would then be given to another child to see whether he or she could reconstruct the polyhedron from the information in the classmate's description. This approach would also allow the teacher to ensure that the children worked with appropriate polyhedrons for this activity rather than the extremely complicated ones that the children tended to build.

Build a Secret Structure

We devised a "build the secret structure" task to see whether we could extend children's thinking about what qualities were required for a complete description. In this activity, done in small groups, the children were asked to build a structure similar to one that we had hidden in a sack. As a group, the students could ask any short-answer questions that they thought would help them with the task.

We wondered whether the children would ask about which shapes shared an edge. We wondered if they would ask about decomposing the structures into smaller units, for example, "Is part of it a pyramid?" The children's questions gave us an indication of which features of polyhedrons were the most important to them. The children routinely asked what shapes were used to make the structure. Most groups also asked how many of each shape was used and how many shapes were used in all. All but one group asked how many faces the polyhedron had. (In this instance, two of its faces were made of three triangles each.) Some groups also asked, "What does it look like when it's flattened out?"

The children found this task difficult to accomplish. In one group, nobody completed the task in the time allotted. In the other groups, at least one child accomplished the task. Even when they were allowed to ask questions, the children still needed to do a good deal of trial-and-error work before they could build the hidden polyhedron. Clearly, the children were not sure what questions to ask to get enough information to build the secret structure successfully.

Although this task was intriguing to the children and engaged them in genuine problem solving, we may have been better off delaying it until the children had more experience to draw on. This task gave children no visual clues, which made it the most abstract of our tasks. The children did not ask about the shapes that shared an edge, because they did not pay attention to edges in their own descriptions. In future units, we would reserve this culminating activity until after the children were able to generate more complete descriptions of their polyhedrons.

Children's Learning

Although the children rarely talked about the shapes that shared an edge or the shapes that connected at a vertex, their work with the polyhedrons did help them develop new perspectives, as shown in Laura's and Anna's final descriptions (see **figs. 3** and **4**). Notice that Laura included a diagram that corresponds with the net of her structure. Anna decomposed her polyhedron into smaller polyhedrons. She mentions that her structure has two pyramids and one cube.

By the end of these activities, all the children were building more complicated structures. We were happy to have given the children appealing building opportunities that many of them, especially the girls, may not have pursued on their own. The children's work provided evidence of their increased spatial understanding as they discovered ways to close the polyhedrons that they built. In addition, their later descriptions of polyhedrons contained more information than their descriptions at the beginning of the project. Even though they did not specifically note edges, these descriptions were evidence of the children's increased use of geometric terminology.

We were excited about the tasks that we had devised, because they served to elicit and build on children's thinking. In our first round of teaching, we were cautious about how much to push students to develop the perspective that we had in mind. When they did not recognize the importance of the shapes that share an edge, we let that idea go, assuming that it was not in line with their thinking. We plan to emphasize the importance of the edges more in the future, spending more time comparing polyhedrons in which this feature might stand out. We now know that children can decompose polyhedrons into smaller polyhedrons and can look for kernels of this kind of thinking and build on it when it emerges. We can also design more tasks that foster this orientation. We anticipate that as we continue to use and revise these tasks, we will gain a clearer idea of the potential in each task and the potential of the children's thinking and that our questioning will improve.

For other teachers who are interested in duplicating our exploration, we encourage you to build on our suggestions, then to critically analyze the results. Our suggestions are based on speculation and have not stood the test of experience. We also encourage you to be patient with yourselves as you engage in questioning your students. Elevating discussions to the level of justification and argumentation envisioned in the Standards is difficult. We are now working to support children in analyzing similarities and differences among polyhedrons to help them begin to see the need for precise terminology. We look forward to learning more about children's emerging spatial understanding as they engage in building and analyzing polyhedrons.

FIGURE 3

Laura's description of her polyhedron

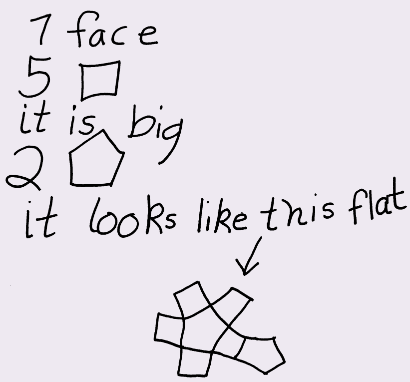
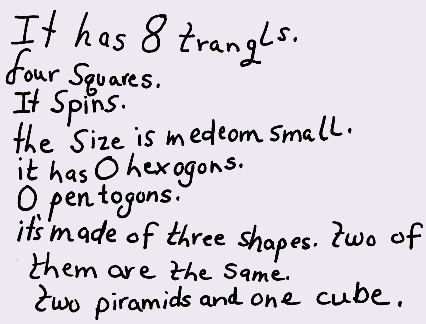


FIGURE 4

Anna's description of her polyhedron



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