What Is Algebra in Elementary School?

Jennifer M. Bay-Williams

Patterns have long been part of early mathematics experiences. The K–4 Patterns and Relationships Standard in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) was replaced in *Principles and Standards for School Mathematics* (NCTM 2000) with a K–12 Algebra Standard. This Standard encompasses patterns, functions, and some topics that are beyond what traditionally was considered to be algebra. However, the word *algebra*, often associated with content covered in a traditional middle school or high school course, can evoke feelings of anxiety and raise questions of appropriateness when discussed in relation to elementary school children. What is algebra in elementary school if it is more than identifying and extending patterns in the early grades yet is not the abstract content of an algebra course?

Principles and Standards identifies four major themes in this Standard for grades K–12: (1) understanding patterns, relations, and functions; (2) representing and analyzing situations; (3) using mathematical models to represent and understand quantitative relationships; and (4) analyzing change in various contexts. The skill that begins in prekindergarten and kindergarten as recognizing and extending patterns develops over the course of elementary school to generalizing problems verbally and symbolically in fifth grade. **Figure 1** lists specific expectations for children in grades pre-K–2 and 3–5. The following paragraphs outline three classroom explorations that help illustrate how these expectations are met in practice.

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Edited by **Jeane Joyner,** *jjoyner@dpi.state.nc.us, Department of Public Instruction, Raleigh, NC 27601, and* **Barbara Reys,** *reysb@missouri.edu, University of Missouri—Columbia, Columbia, MO 65211. This department is designed to give teachers information and ideas for using the NCTM's* Principles and Standards for School Mathematics *(2000). Readers are encouraged to share their experiences related to* Principles and Standards *with* Teaching Children Mathematics (TCM). *Please send manuscripts to "Principles and Standards,"* TCM, *1906 Association Drive, Reston, VA 20191-9988.*

Card Patterns

First graders enjoy working with patterns. Their curiosity and creativity are a good match for the algebraic thinking fostered at this level. In this exploration, the teacher created several decks of cards, each with a different numeric pattern. Some of the patterns were repeating (ABCABC), some were growing (counting up by threes or down by fives), and some were alternating (up two, down five, up two, down five). The cards for one pattern were placed in a row in front of the students with only one card turned faceup. From that card, the students made predictions about what they thought the next number might be.

One pattern started with a 4 and a 6. The students thought that the next value would be an 8 and were surprised when it was a 4. Once the students saw that the first three numbers were 4, 6, 4, they predicted that the next value would be 6. One student remarked that this pattern had to be an AB pattern. In this instance, the students were correct, but other possibilities existed. For example, the pattern could have been 4, 6, 4, 8, 4, 6, 4, 8 (ABACABAC) or 4, 6, 4, 4, 6, 4, 4, 6, 4 (ABAABA). Students in primary grades must learn to identify the start and end of the unit of a pattern and be aware that they may need to see many numbers before they can conclusively determine a pattern.

Figures 2 and **3** show students analyzing a pattern in which a 40 was first uncovered from the middle of the sequence. When asked what number

FIGURE 1

Algebra expectations for students in grades pre-K–2 and 3–5

Algebra Standard

Instructional programs from prekindergarten through grade 12 should enable all students to—	**Pre-K–2 Expectations** In prekindergarten through grade 2 all students should–	**Grades 3–5 Expectations** In grades 3–5 all students should–
Understand patterns, relations, and functions	• sort, classify, and order objects by size, number, and other properties; • recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another; • analyze how both repeating and growing patterns are generated.	• describe, extend, and make generalizations about geometric and numeric patterns; • represent and analyze patterns and functions, using words, tables, and graphs.
Represent and analyze mathematical situations and structures using algebraic symbols	• illustrate general principles and properties of operations, such as commutativity, using specific numbers; • use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.	• identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers; • represent the idea of a variable as an unknown quantity using a letter or a symbol; • express mathematical relationships using equations.
Use mathematical models to represent and understand quantitative relationships	• model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.	• model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.
Analyze change in various contexts	• describe qualitative change, such as a student's growing taller; • describe quantitative change, such as a student's growing two inches in one year.	• investigate how a change in one variable relates to a change in a second variable; • identify and describe situations with constant or varying rates of change and compare them.

From *Principles and Standards for School Mathematics* (NCTM 2000, p. 394); all rights reserved

would come next, the students offered many responses.

Student 1. I think the next number is 50 because it is counting by tens.

Student 2. I think it is 80; you know 40, 80, 120.

Student 3. I think it is 42—counting by twos.

Student 4. I think it is counting by elevens.

Teacher. Then what would the next number be?

Student 4. Umm, 51.

After the students shared their predictions, one student flipped over the next card, which was a 30. Most students thought that the next card would be 20, predicting that the pattern was counting down by tens. One student said that she thought the next card would be 40, stating that the cards followed another ABAB pattern. The third number uncovered was 20. Although the students agreed that the pattern seemed to be counting down by tens, they reported that other possibilities still existed and that they did not have enough numbers to know for sure.

Another set of cards showed the pattern 18, 23, 28, 33, 38, 43, 48. The students identified the pattern as +5. After the students had uncovered all the cards, the teacher asked them to look for patterns in the numbers. The students noted that the endings alternated 8, 3, 8, 3, 8 and that the pattern for the tens place was 2, 2, 3, 3, 4, 4, except for 18 because 13 was not on a card.

The algebra content in this exploration is found in the students' describing and extending numeric patterns. Once the students identified the pattern unit, they were able to predict what card was several cards away without uncovering every card in the sequence. They also described the repeating and growing patterns that were generated. In addition, for simpler patterns, such as 1, 3, 5, 7, 9 and 10, 20, 30, 40, 50, 60, the students analyzed patterns in the numbers themselves.

FIGURE 2

First graders turning up a middle card in a pattern

Pattern-Block Patterning

In a combined fourth- and fifth-grade class, the students spent two weeks studying growing patterns with pattern blocks. In one of the early patterns,

FIGURE 3

First graders further analyzing the pattern

the first design had two parallelograms, the second had four parallelograms, the third had six parallelograms, and so on. The students created a table of the number of parallelograms needed for each design to continue the pattern. From the table, the students studied patterns to predict the number of pattern blocks that would be needed to build the twentieth and thirtieth designs in the pattern. In the end, the students were asked to describe the general case and to record it symbolically. For these examples, the students concluded that the general pattern was to double the term to get the number of parallelograms. The teacher showed that this generalization could be recorded in symbols as $2 \times n$. On another day, the students looked at a pattern that started with three triangles and grew by three with each new design. The students generalized that for this pattern, the number of pieces needed was three times the design number; the class recorded this idea symbolically as $3 \times n$. In reflecting on the parallelogram and triangle patterns, the students noticed that the starting number of pieces was the same as the increment with each new design and was also the multiplier for the variable in the general case.

At the end of the week, the students were given a new pattern that started with three hexagons. Each subsequent term increased by two hexagons, which were added to the right side to create a wall. As a class, the students made a chart to record the number of hexagons used each time (see **fig. 4**).

This pattern proved to be very difficult for the students to generalize. They noted that for each new design, they needed two more hexagons than for the previous design. They also noted that the table increased by two each time. These realizations allowed the students to predict the number of hexagons needed for the next design in the sequence but not to predict the number of hexagons that would be needed for the twentieth, thirtieth, or *n*th design. To make these generalizations, the students had to discover the horizontal pattern in the table by comparing the design number with the number of hexagons needed. In groups, the students discussed and tested various hypotheses (see **fig. 5**). The resulting dialogue involved important algebraic reasoning. At one table, the dialogue was as follows:

Student 1. I think it will be times 3 because the pattern starts with 3 and that's how the others have worked.

Student 2. I think it will be times 2 because it's going up by twos.

Student 3. It can't be times 2 because the answers are all odd; it has to be times 3.

Student 2. But times 3 doesn't work. Look—if you plug it in here [pointing to the third term], you get 9, not 7, which is what it is.

After exploring further and building more designs, the students came up with two generalizations that worked when they tested them. One student explained that the pattern went as follows: remove one hexagon from the first design, then multiply by 2, then add the one hexagon back on in the end. Several groups explained that the pattern was to double the number of hexagons, then add 1. These students wrote the pattern generalization as $2 \times n + 1$. One group recorded this generalization as $1 + 2 \times n$, explaining that the first design had an extra piece and that thereafter, the pattern grew by two each time. One student, referring to the table, explained that the total number of hexagons needed was the design number plus the design number that

FIGURE 4

A student records data from the hexagon pattern in a table.

FIGURE 5

These students try to determine the general case for the hexagon pattern.

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was one ahead of it. With assistance from the teacher, the class generated the symbolic notation $n + (n + 1)$ for that description.

This task required students to extend a geometric pattern, describe that pattern, generalize it, and represent it geometrically. In addition, the students represented the pattern by drawing it, recording it in a table, describing it in words, and for many students, writing the algebraic expression for the pattern in symbols. Moving between representations is important in making sense of patterns and functions. Different students relied on different representations to make sense of the pattern. The students analyzed change in the context of pattern-block designs. In doing so, they explored how the change in one variable (the design number) related to another variable (the number of hexagons needed). Students in grades 3–5 are also expected to identify the commutative, associative, and distributive properties. In the three expressions generated above, $2n + 1$, $1 + 2n$, and $n + (n + 1)$, the students could see that each representation is correct because each will generate the number of hexagons needed to create the design. Asking students to explain why these three expressions result in the same answers encourages them to apply the commutative and associative properties.

Creating and Analyzing Color-Tile Patterns

In a fifth-grade class, the students were asked to create their own designs with color tiles, record the data in tables and graphs, and use that information to determine the general rules for their designs. These students had many previous experiences with generalizing patterns that had been shown to them, but creating their own patterns resulted in new learning about what constitutes an algebraic pattern. Most students' first designs were so complicated that they could not figure out what the pattern might be or whether a pattern existed. In further explorations, the students generated patterns that grew by some constant rate.

Figure 6 shows two boys working on a pattern in which each stack of tiles grew by one more than the previous stack. In **figure 7,** the pattern grew in an alternating fashion. The students recorded and graphed both the step function, that is, the number of new tiles needed for just one step, and the total number of tiles needed. Developing symbolic expressions for such formulas is difficult and beyond these students' abilities; however, the students were able to describe the patterns of their designs and to verbalize the generalizations. The students were then asked how many tiles they would need to generate the tenth and twentieth designs in the sequences and at what step they would need more than 100 tiles for the total design.

As they had done with the hexagon task, these students were analyzing geometric patterns and making generalizations. They analyzed the patterns in tables, graphs, and words. The students described whether their patterns grew at constant rates or varying rates. For example, in the pattern illustrated in **figure 6,** the boys debated whether the rate of their pattern was varying or constant, finally concluding that the rate was varying because it varied by one with each step. The girl who designed the pattern in **figure 7** explained that her rate was constant, even though it alternated. In creating patterns, the students had to think more deeply about what elements define a pattern. Finally, the students used the information in their tables or graphs to draw conclusions.

FIGURE 6

These students analyze a pattern that they created and try to generalize it.

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Algebra in Pre-K–5

The three classroom episodes illustrate appropriate tasks that develop important algebraic concepts for elementary school students. These examples also support three important themes that are related to algebra and emphasized in *Principles and Standards.* These themes are summarized below.

Algebra is closely related to other content strands. Algebra builds on students' experiences with number. In each of the examples discussed, the students used their knowledge of skip counting and whole-number operations to look for patterns. At the same time, in revealing patterns that increase by 3 or 4, the algebraic explorations supported the students' understanding of number. Data analysis and geometry are also important in developing algebraic thinking. Students can collect data and look at patterns in the data. In the latter two explorations, the students examined geometric patterns, analyzed data in tables and graphs, and drew conclusions.

A second important theme related to algebra is that patterns and functions should be represented in a variety of ways. Words, tables, graphs, symbols, and diagrams can all be used to analyze patterns and functions. Incorporating several of these forms in the same exploration enables students to see relationships among the representations and to move flexibly among the different forms. In addition, because certain representations make more sense to some students, using different representations enables more students to understand the ideas presented. In the color-tile task, some students were able to see the patterns only by looking at the designs that they had built. Other students used their tables or looked at the linear growth in their graphs.

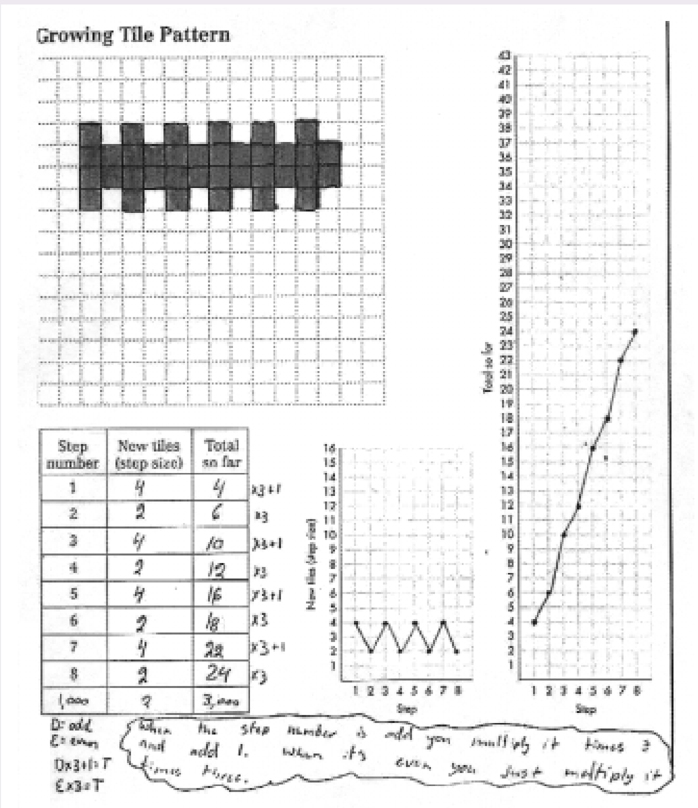
Finally, understanding the commutative, associative, and distributive properties is important for children. Although young children do not need to know the names for these properties, they do need to know that 4 + 7 has the same result as 7 + 4. Students adding 8 + 5 might decompose the problem to 8 + 2 + 3, then add 2 to 8 to get 10 and add 3 more. As illustrated in the hexagon problem, upper elementary school students must know when one expression is equal to another expression. Note that the emphasis is not on identifying and illustrating the properties but on knowing that these properties hold true and developing the ability to apply them flexibly when appropriate.

Educators must provide algebraic experiences that are developmentally appropriate and grow in sophistication for students in grades pre-K–5. What algebraic concepts should be taught in the elementary school years has not always been clear. The classroom episodes described in this article are intended to provide some insight into the nature of algebra at various levels. More support can be found in NCTM's *Navigating through Algebra* books (2001a, 2001b), resources that were developed to elaborate on the algebra strand in *Principles and Standards* and that offer lesson ideas, assessments, and a thorough discussion of the mathematics content.

Students in grades pre-K–5 enjoy studying patterns and figuring out how they work, whether the patterns are geometric or numeric, repeating or growing. In these years, students learn how to describe and model situations in their world. Algebraic experiences in elementary school are essential in building the thinking that is "an important precursor to the more formalized study of algebra in the middle and secondary schools" (NCTM 2000, p. 159).

FIGURE 7

Liz's representations of her pattern in a drawing, a table, graphs, and words



References

National Council of Teachers of Mathematics (NCTM). *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: NCTM, 1989.

———. *Principles and Standards for School Mathematics.* Reston, Va.: NCTM, 2000.

———. *Navigating through Algebra: Prekindergarten–Grade 2.* Reston, Va.: NCTM, 2001a.

———. *Navigating through Algebra in Grades 3–5.* Reston, Va.: NCTM, 2001b. ▲